Vibration pumping of mdof structures using optimised multiple dynamic absorbers

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ABSTRACT

The concept of energy pumping is an innovative dynamic phenomenon; it gives rise to new generation of dynamic absorbers. Theoretical studies and feasibility tests are necessary for better understanding of their dynamic behaviour and to be applied on real structures or machines.

In this paper, numerical evidence is firstly given for the passive and broadband targeted energy transfer in the case of a linear system under shock excitation with Multiple Dynamic Absorbers or Nonlinear Energy Sink (NES). Secondly, it is shown that many NES absorb shock energy in only way and dissipate this energy locally, without "spreading" it returns to the linear system. The numerical results of optimisation in the case of NES linked to a linear beam are compared to Tuned Mass Dampers (TMD) linked to the same beam.

***Keywords**: Energy pumping, dynamic absorber, Cubic non-linearity, nonlinear energy sink, tuned mass damper, Optimisation.*

1 INTRODUCTION

The phenomenon of energy pumping is an irreversible transfer of energy from a main structure to a secondary structure, as dynamic absorbers called nonlinear Energy Sink (NES) are linked to the main structure. In previous works [1-3], it has been shown that essentially nonlinear oscillators attached to linear discrete structures that act as broadband passive absorbers of vibration energy. Particularly, it has been shown that transient resonance captures of the transient dynamics may initiate one way, irreversible targeted energy transfer from a linear (main) subsystem to a local essentially nonlinear attachment, which acts, in essence, as nonlinear energy sink (NES). In this work, we propose to optimise the portion of energy damped by the linear and nonlinear dampers linked separately to a beam. The design parameters to optimise are mass ratio, linear and nonlinear stiffness and damping ratio of the dynamic absorbers. For this multi-objective optimisation we calculate the Pareto solutions using NSGA algorithm (Non-dominated Sorting Genetic Algorithm) [4, 5].

2 DYNAMIC EQUATIONS OF MOTION OF MDOF SIMPLY SUPPORTED BEAM

The studied structure consists of an impulsively forced simply supported damped linear beam, comporting multiple dynamic absorbers (Figure 1). This system was introduced in [1] with only one NES.

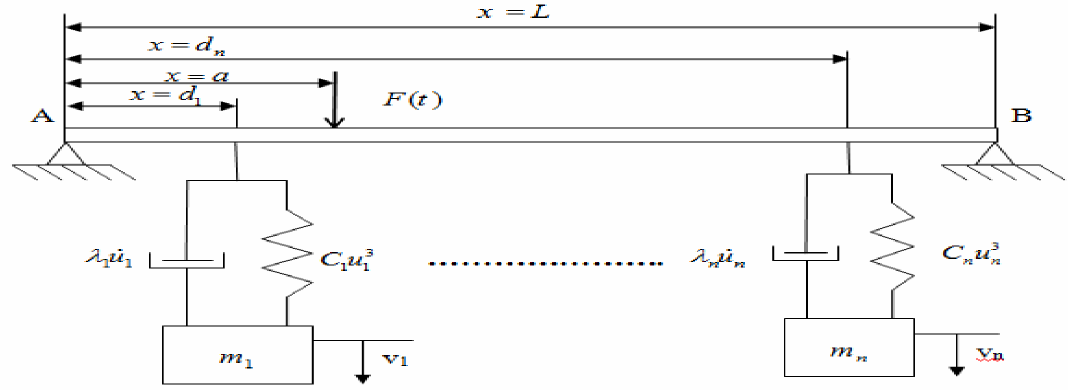


Figure 1. Simply supported beam with Multiple Dynamic Absorbers

The beam finite element model is based on the Euler-Bernoulli theory.

$$\begin{cases} EI\dfrac{\partial^4 y(x,t)}{\partial x^4}+\lambda\dfrac{\partial y(x,t)}{\partial t}+\rho A\dfrac{\partial^2 y(x,t)}{\partial t^2}+\displaystyle\sum_{i=1}^{n}\left(\left\{C_i[y(d_i,t)-\mathrm{v}_i(t)]^3+\lambda_i\left(\dfrac{\partial y(d_i,t)}{\partial t}-\dot{\mathrm{v}}_i(t)\right)\right\}\delta(x-d_i)\right)=F(t)\delta(x-a) \\ m_i\ddot{\mathrm{v}}_i(t)+C_i[\mathrm{v}_i(t)-y(d_i,t)]^3+\lambda_i\left[\dot{\mathrm{v}}_i(t)-\dfrac{\partial y(d_i,t)}{\partial t}\right]=0 \qquad i=1,2.....n \end{cases} \tag{1}$$

The finite element model for the beam with NES is written as follow:

$$M\ddot{y}(t)+B\dot{y}(t)+K(y)y(t)=F(t) \tag{2}$$

Where M, K and B are respectively the mass, stiffness and damping matrix.

The energy dissipated by the NES and the TMD at time t is defined by the ratio:

$$\eta(t)=\frac{E_{NES/TMD}(t)}{E_{inj}}=\frac{\displaystyle\int_0^t\sum_{i=1}^{n}\lambda_i(\dot{\mathrm{v}}-\dot{\mathrm{y}})^2_{x=d_i}\,d\tau}{\displaystyle\int_0^T F(\tau)\dot{\mathrm{v}}_{(x=a)}\,d\tau} \tag{5}$$

Where F , λ_i , $\dot{v}$ and $\dot{y}$ represent respectively the impulse force, damping ratio and the velocity at $x = d_i$ (location of the NES and TMD).

3 OPTIMISATION STUDY

In this section, we propose the multi-objective optimisation of the linear and nonlinear dynamic dampers (TMD, NES). The NSGA genetic algorithm is used to explore the design space and exploit the whole solution of the Pareto front [4]. For this study the multi-objective optimisation is defined in the following form:

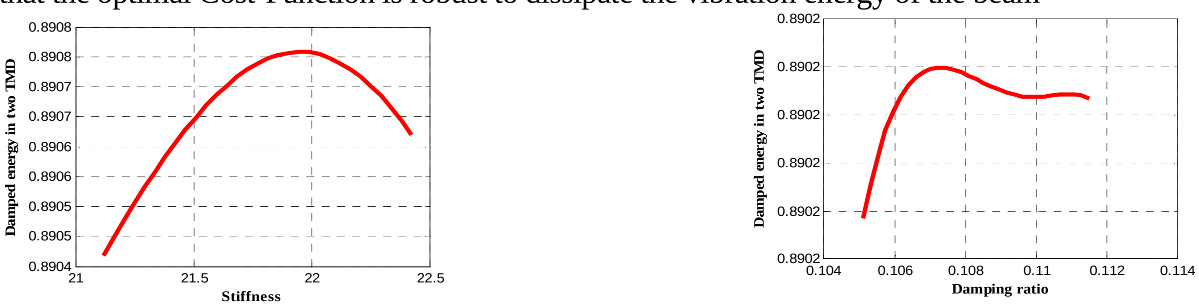
$$\begin{cases} \underset{x}{Max}(F1) = \eta \quad where\ x \in [m_i, \lambda_i, k_i, C_i] \quad i = 1, 2, \ldots n \\ \underset{x}{Min}(F2) = \dfrac{\sum_{i=1}^{n} m_i}{M} \end{cases} \tag{6}$$

Where m_i, λ_i, k_i, C_i are the coefficient of the mass, damping and stiffness of the TMD or the NES; M is the primary structure mass.

3.1 Optimisation of the beam with two TMD

In the first case of this optimisation, we consider the beam defined in section2. The parameters of the two TMD are identical: $k = 27.215 N/m^3$, $m = 0.3kg$ and $\lambda = 0.057 N.s/m$. These TMD are coupled in the beam at location ($d_1 = 0.25m$ and $d_2 = 0.8m$). The variation levels of the design parameters are 20% for (m, k) and 90% for λ. As a results the maximum of the F_1 is 89.04% for the minimum of $F_2 \leq 0.4kg$ corresponding to the design parameters $k_{opt} = 21.7720 N/m^3$,

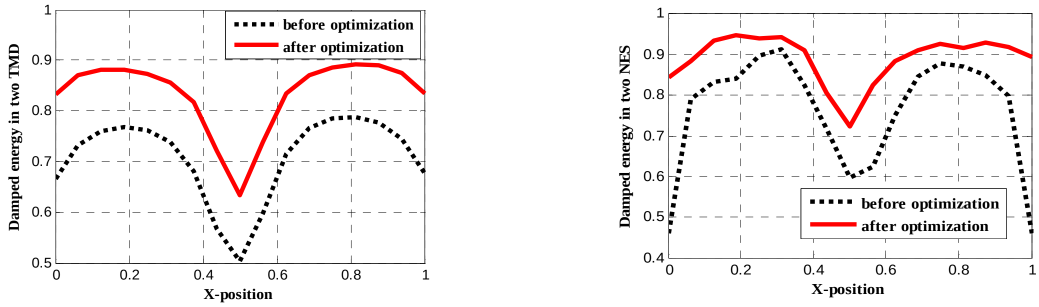
$m_{opt} = 0.2926kg$ and $\lambda_{opt} = 0.1083 N.s/m$. Then, we examine the influence of variation of the optimum values of stiffness and damping ratio. The figure2 illustrates this variation. We note that a variation of 3% of k_{opt} and λ_{opt} leads to a variation of 0.03% of F_1 . This low variation shows that the optimal Cost-Function is robust to dissipate the vibration energy of the beam



(a) (b)

Figure 2. Variation of dissipated energy ratio in two TMD: – (a) variation of stiffness – (b) variation of damping ratio

Finally, we examined the efficiency of the two TMD to dissipate the energy with different locations d before and after optimisation. Figure3a shows this variation, we note that the energy ratio absorbed by the two TMD before optimisation is $\eta = 89\%$ and after optimisation it becomes $\eta = 78\%$ at two locations of TMD : $d = 0.2m$ and $d = 0.8m$.



a- Dissipated energy versus X-position (TMD case)

b- Dissipated energy versus X-position (NES case)

Figure 3 : Dissipated energy with optimised dynamic absorbers

3.2 Optimization of the beam with two NES

In this section, we consider the same beam with a two NES at different locations (Fig.1). The following parameters of the two NES are identical: $C_1 = C_2 = C = 1322\,N/m^3$, $m_1 = m_2 = m = 0.1kg$ and $\lambda_1 = \lambda_2 = \lambda = 0.05Ns/m$. We consider the same variation levels of the design parameters defined in the previous section except that the variation of C is 50%. As a results, the maximum of F_1 is 95% for $F_2 = 0.2kg$ and the optimum value obtained by optimisation of design parameters are: $C_{opt} = 1983\,N/m^3$, $m_{opt} = 0.08kg$ and $\lambda_{opt} = 0.095\,Ns/m$. Now, we examine the robustness of the optimum values (C_{opt}, λ_{opt}) by variation of 3%. We note that the variation of C_{opt} and λ_{opt} for this percentage (3%) causes a small variation of 0.04% of the cost-function F_1. So, these optimum values are robust to dissipate the maximum of energy from the beam. Then, we examine the efficiency of two NES to dissipate the shock energy before and after optimisation by varying its location on the beam. Figure3b illustrates the energy ratio dissipated by two NES for different positions on the beam. We note that this ratio reaches 95.40% after optimisation with two positions $d_1 = 0.2m$ and $d_2 = 0.8m$, whereas before optimisation this ratio is $\eta = 91\%$. As a conclusion, the variation of the position of the two NES affects its efficiency on the middle of the beam and we note that the two NES design are more efficient than two TMD design.

4 CONCLUSION

In this paper, we present a multi-objective optimisation of the dynamic absorbers (TMD and NES). In the first step of optimisation, we consider beam with two TMD the dissipated energy ratio after optimisation is 85%. On the other hand, this ratio reached 95% in the case of the beam with two NES. The robustness study with two TMD and two NES shows that the variation of the design parameters around the optimal values does not affect the dissipated energy ratio. Finally, the variation of the position of the two TMD and two NES affects its efficiency on the middle of the beam.

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