Coupling of high quality factor optical resonators

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Abstract. We improve theoretically and experimentally the problem of the coupling between a high Q-factor resonator and its external coupler. We have observed oscillations of ringing induced by the sweeping of the excitation frequency of an active micro sphere. Thanks to this approach, quality factor of an optical resonator was measured and we obtained $Q = 5.8 \times 10^8$.

1. Introduction.
Optical resonators with high quality factor (Q-factor) present a great interest for several applications in optoelectronic [1], metrology or fundamental physics [2, 3]. For instance they can be used in optical filtering, all optical switching, low threshold non-linear optics, etc. Main results have been obtained since 1996 [4]. Recently results have been obtained with mono-crystalline such as MgF$_2$, Quartz or CaF$_2$ whispering gallery mode (WGM) resonators [5-9]. Rare earth element doped micro spheres are tiny lasers which intrinsically offer narrow line width emission [10]. A key feature of the characterization of the resonators is to perform a good coupling. Performing this characterization is quite challenging because it is a necessary but difficult technique. That’s why we improve theoretically and experimentally the problem of the coupling between a high Q-factor resonator and its external coupler. They are characterized by different methods such as slow and fast frequency sweeping [11]. Resonators can be coupled to tapered fibres or prisms. We focus here on characterization techniques of passive optical resonators or active (Erbium doped ZBLALiP [12, 13]) WGM resonators in terms of Q-factor determination.

Figure 1 shows schematically the WGM micro resonator and its access line. It is possible to isolate each resonance, although the resonator is inherently multi mode. We can work with each resonance separately. The studied resonator is then considered single mode.

A mode with an amplitude $a(t)$ is characterized by the time life of the field $\tau$ limited by losses, by the coupling rate of the access line which is proportional to $1/\tau$ and by the angular frequency of the resonance $\omega_0$. The amplitude of the input signal is noted $s_{in}(t)$, we can then calculate the amplitude of the output signal $s_{out}(t) = -s_{in}(t) + \sqrt{\frac{2}{\tau}} a(t)$ by integration:

$$\frac{da}{dt} = \left(j\omega_0 - \frac{1}{\tau}\right)a(t) + \sqrt{\frac{2}{\tau}}s_{in}(t)$$

In equation (1), $\tau = (1/\tau_0 + 1/\tau_e)^{-1}$ is the time life of the field in the cavity related to factor $Q = \omega_0^2 \tau / 2$. We assume that input signal is harmonic $s_{in}(t) = S_{in} e^{j\omega t}$ and output signal is $s_{out}(t) = S_{out} e^{j\omega t}$. As a consequence, the transfer function is given by:

$$T(\delta) = \left| \frac{s_{out}}{s_{in}} \right|^2 = \left( \frac{1/\tau_e - 1/\tau_0}{1/\tau_e + 1/\tau_0} \right)^2 + \frac{4\pi^2 \delta^2}{(1/\tau_e + 1/\tau_0)^2 + 4\pi^2 \delta^2}$$

(2)
In this expression $\delta$ is the frequency mismatch given by $\omega = \omega_0 + 2\pi \delta$. Different configurations are possible according to the relative value of $\tau_e$ and $\tau_0$. If $\tau_0 \to \infty$ then resonator has no losses, so $T(\delta)=1$ as illustrated on figure 1(b). But if we suppose that $\tau_e = \tau_0$, then the resonator is in critical coupling and transmission at the resonance $T(0)$ becomes zero. And if $\tau_0 < -\tau_e$, resonator presents some gain (even if it stays less than the LASER threshold) and it behaves like an amplifier, then $T(0) > 1$ [14].

![Figure 1. Scheme of the micro resonator and its access line.](image)

Amplification measurements were performed in Erbium doped ZBLALiP micro spheres. Diameters of the micro spheres are between 50 µm and 150 µm. Theses spheres are obtained by fusion in a microwave inductively coupled plasma [10]. Usual measurement method consists in sweeping the frequency of a Laser emitting around 1550 nm, in order to obtain a description of one resonance for the micro sphere. Simultaneously, this micro sphere is pumped by a laser at 1480 nm emitting in opposition of propagation as described in Figure 2.

![Figure 2. Setup for measuring the selective gain.](image)

Thanks to the pumping, the sphere delivers a Laser signal for one or several resonances far enough from the studying resonance: it helps to stabilize the gain. Optical Spectrum analyzyr (OSA) helps for the control the micro sphere Laser emission. To illustrate how this technology can be applied, laser frequency is swept slowly enough to stay in stationary approximation, to describe thin resonance as illustrated in Figure 3.
Our approach is different. The analysis begins by sweeping very quickly the frequency of the Laser. This process is analogous to what occurs when the speed is slow, but provides definitely an easier and unquestionably a precise determination of the Q-factor. This approach is implemented by utilizing the following process. This allows to work in the ringing regime of the micro sphere. It has already been demonstrated that it is possible to determine characteristic parameters of the micro sphere [11, 15] if we compare the profile of the ringing oscillations to an analytical model obtained thanks to Eq.(1) and to an incident field like $s_{n}(t) = S_{n} e^{j \omega_{r} t} e^{j \delta t}$. The sweeping speed of the Laser $V_{S}$ is often difficult to obtain precisely. But it is directly obtained by the method describes above. The results are illustrated in Figure 4, where two time profiles are obtained on the same resonance, with two different speeds (the second one is two time more quickly) for sweeping the Laser. While the deduced speeds are in the good ratio, adjustment provides almost the same results for $\tau_{0}$ and $\tau_{e}$. Observed differences for $\tau_{0}$ and $\tau_{e}$ are mainly due to the fluctuations of the taper position. It has an influence on the signal coupling, and also on the pump. When performing the adjustment from the obtained ringing oscillation profiles, it has to be underlined that we deduce a quality factor as high as $Q = 4.4 \times 10^{8}$. The analysis proceeds by matching the theoretical and experimental curves. Q-factor estimation may be quickly obtained, and the efficiency of such a fact sweeping is really good. This is the desired result, which gives great insight into the resonator coupling and Q-factor estimation problem.

Figure 3. Theoretical spectra obtained for $3 \tau_{0}$.

Figure 4. Obtained ringing oscillation profiles for two frequencies $F$ driving the sweeping of the tunable Laser, $P_{S} = 125$ nW, $P_{P} = 7$ mW. (a) – left: $F = 400$ Hz, these values are deduced from the adjustment : $\tau_{0} = -40.9$ ns, $\tau_{e} = 38.8$ ns, $VS = 2.2$ MHz/µs, $Q = 4.4 \times 10^{8}$ and $T(0) = 31.2$ dB. (b) –
right: \( F = 800 \) Hz, also deduced from the adjustment: \( \tau_0 = -39.2 \) ns, \( \tau_e = 37.6 \) ns, \( V_S = 4.1 \) MHz/\( \mu \)s, 
\[ Q = 5.8 \times 10^8 \text{ et } T(0)=33.9 \text{ dB}. \]

4. Coupling with passive resonator and integration into an oscillator.

An optoelectronic oscillator (OEO) with an optical resonator coupled to an optical fiber, delivers a free spectral range microwave signal [1]. To implement this OEO in terms of miniaturization, it is possible to use a high WGM resonator with magnesium fluoride or calcium fluoride for microwave photonics applications. We have fabricated and characterized 5 mm diameter toroid MgF₂ and 12 mm CaF₂ resonators with a good surface roughness (1 nm). Quality factor better than \( 3.4 \times 10^8 \) for magnesium fluoride [7] and \( 1.4 \times 10^8 \) for calcium fluoride were measured by cavity ring-down measurements. Even if an optical line can be used as a frequency selective component in the OEO loop, the use of the mini-resonator leads to a significant reduction of the oscillator size. Further work consists in measuring the phase noise on dedicated instruments developed at the laboratory [16, 17]. We can also reduce or eliminate sensitivity to vibration by using a refractive index adapter gel. Q factor of resonators can still be improved by a better polishing of the crystal coupled to other treatments.

5. Conclusion.

The work performed here is significant in the sense that it appears that the problem of the coupling between a high Q-factor resonator and its external coupler has been theoretically and experimentally improved. The key is to recognize that oscillations of ringing induced by the sweeping of the excitation frequency of an active micro sphere are observed. Measured apparent gain is approximatively 27dB (up to 33.9 dB in stationary regime) in a 330 kHz band corresponding in the best case to a high quality factor \( Q = 5.8 \times 10^8 \). All of these approaches have been developed to address high quality factor resonator characterization. Addressing the combination of resonator polishing with other treatment, such as thermal annealing, to improve the surface roughness, is still an active area of research to achieve higher Q factors.

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References.