A distributed model predictive control scheme for reducing consumption of hybrid fuel cell systems

Sébastien Mariéthoz
Automatic Control Lab.
ETH Zürich, Switzerland
Email: mariethoz@control.ee.ethz.ch

Olivier Bethoux and Mickaël Hilairet
LGEP/SPEE Labs., CNRS SUPELEC, Universités
Pierre et Marie Curie-P6 and Paris Sud-P11, Paris, France
Email: olivier.bethoux@lgep.supelec.fr

***Abstract*—The paper introduces a hierarchical structure for the finite time constrained optimal control of a system with a fuel cell and a supercapacitor storage unit. The supercapacitor storage unit is employed to improve life time, reduce hydrogen consumption of the fuel cell, increase peak power and allow for some regeneration from the load. The three control layers regulate the supercapacitor storage unit charge, the DC bus voltage level and the currents of the fuel cell and supercapacitor storage unit. They are all based on explicit linear offset-free model predictive control, where the optimal control law is computed off-line and stored in a look-up table.**

***Index Terms*—Fuel cells, Supercapacitors, Predictive control, MPC, Optimal control, Energy management, Energy efficiency, Energy storage, Batteries, Distributed control**

I. Introduction

Global environmental concerns and the ever-increasing need for electrical power generation [1] increase interest in distributed generation. Fuel cell is an attractive pollution-free energy source [2] that offers efficient and quiet conversion of hydrogen, which is a high density energy carrier ($120\,\mathrm{MJ\,kg^{-1}}$) [3]. Moreover, hydrogen can be generated from renewable resources via water electrolysis (solar photovoltaic, wind turbine, water turbine) or methane gas reforming (issued from biomass energy). Owing to their high volumetric and gravimetric specific power densities, quick start-up and high-energy efficiency, proton exchange membrane (PEM) fuel cells are presently the most promising and established technology for stationary and mobile applications as well as for transportation [4].

Performance, durability and failure rate of fuel cells widely depend on operating conditions [5]–[7]. It is necessary to guarantee the desired electrochemical mechanisms at the cathode side (the reduction of O_2 to H_2O) and the anode side (the oxidation of H_2 to H^+ protons). To this end, the reactants fluxes at the catalyst layer have to be accurately adjusted to the current, which drives the gases consumption. Owing to the slow time constants of transport phenomena and air compressor, load current demand has to be smoothed with slope constraints. Consequently, in many applications such as transportation, FC must be assisted by a storage unit capable of providing the fast power demand, leading to a hybridized electricity source [8], [9]. In such hybrid configurations both the selection of the converter architecture and control structure play an important role in the system performance [10].

In model predictive control (MPC), the control objectives are formulated as a cost function to minimize, while the system constraints are formulated as inequalities to respect. MPC has already been applied to the internal control of fuel cells [11]–[14] and to the control of hybrid systems comprising a fuel cell and a storage unit [15], [16]. In these systems, a battery or a supercapacitor storage unit is employed as buffer to decouple the load and the fuel cell. In [15], the authors use hybrid systems to model the rule based controller of a lithium ion battery and switching decisions. They use this hybrid model to derive a centralized MPC scheme In [16], the authors use the fuel cell to regulate the bus voltage and they use the optimum power point as set point. The selected control structure forces the fuel cell to react quickly.

The present paper introduces a few innovations compared to previous works. First, it shows that the lower consumption is obtained by *switching* between zero and the optimum power point (OPP) for average powers below the OPP and by tracking the *average* power for average powers above the OPP. It proposes a distributed MPC structure where the supercapacitor storage unit regulates the bus voltage, while the fuel cell regulates the supercapacitors. Compared to previous works, adopting ramp constraints for the fuel cell and maximum and minimum constraints for the supercapacitor unit allows to exploit optimally the available energy in the supercapacitor, which in turns reduces the overall consumption.

II. Proposed control structure and strategy

A. System structure

The system structure comprises a storage unit and a fuel cell connected to a common DC bus through two converters as illustrated in Fig. 1. This converter structure appears to be an efficient solution to provide the requested flexibility and it can ensure load demand is always satisfied, while guaranteeing safe operation for each component [10]. The main role of the storage unit is to provide an energy buffer that allows to partially decouple the fuel cell and the load powers. The load may temporarily be regenerative as the fuel cell and the load power need only to match on average. Supercapacitors (SCs) seem to be one of the most suitable components to serve as storage unit due to their very high power density, efficiency, life time and energy density.

978-1-4673-2420-5/12/$31.00

In this work, we assume that the internal control of the fuel cell is provided by the fuel cell manufacturer. We investigate the optimal split of the power between the fuel cell and the storage unit.

B. *Fuel cell losses*

The difference between the net energy supplied to the load and the chemical energy provided by the hydrogen tank is due to two different phenomena illustrated in Fig. 2

- First, FC converts the chemical energy of fuel directly into DC electrical energy simultaneously producing heat due to entropy variation (reversible process) and irreversible processes.
- Second, the stack current is partially used to feed the different ancillary devices that enables a good FC operating points (air compressor, H2 valve, FC cooler, FC monitoring, ...). As a result, the FC system self-consumes auxiliary power even at no load. Hence, the real FC system efficiency reveals a maximum point.

C. *Optimal operation of hybrid fuel cell system*

Since there are infinitely many fuel cell power profiles that satisfy the load and the fuel cell constraints, the problem of finding a suitable power profile can be formulated as an optimization problem where the main objective is to reduce the hydrogen consumption while operating the fuel cell safely. In order to operate the fuel cell safely, the fuel cell current needs to stay within its bound. The number of transitions of the fuel cell between its on and off states needs to be limited as every transition to the on state corrodes the platinum support, which dramatically degrades the fuel cell performance and its durability. In this section, we identify 3 modes of operation with the help of the loss characteristic depicted in Fig. 3.

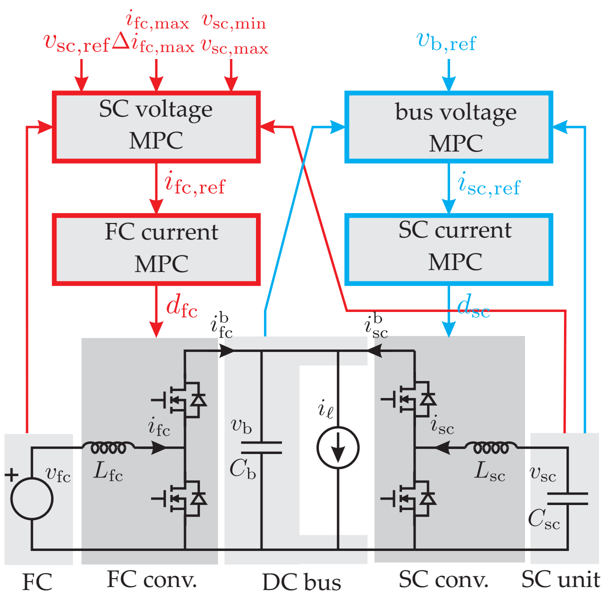

Fig. 1. System overview. Three units connected by two buck converters. Four control loops distributed in two hierarchical groups.

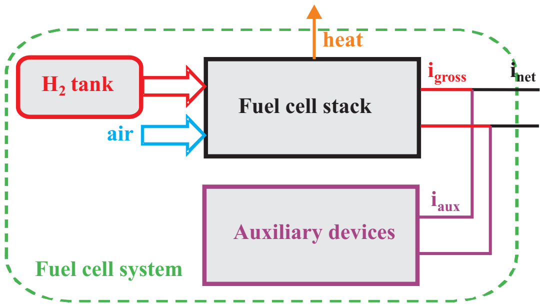

Fig. 2. Losses in fuel cell system

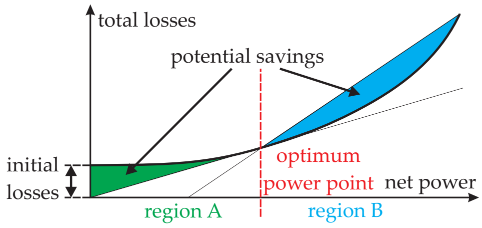

Fig. 3. Sketch of the fuel cell losses as the function of the output power. Two regions delimited by the point of highest efficiency are identified.

The losses are convex with respect to the output power and there is an offset that indicates that when the fuel cell is on, there are losses and self consumption even when the net power is zero. When the fuel cell is off, the losses fall to zero, which causes a discontinuity in the loss characteristic. We will consider separately the cost of switching on the fuel cell. Since there is an offset in the loss characteristics, the maximum efficiency is not obtained at zero. It is obtained where the line crossing the origin is tangent to the loss characteristics as can be seen in Fig. 3 at the point indicated as optimal power. Beyond this point the efficiency decreases. The remarkable property of the optimal power point is that it separates two regions where the optimal ways to operate the fuel cell are radically different.

1) Operation below optimal power point: Below the optimal power point, losses can be reduced by alternating between zero power, fuel cell off (leading to no losses) and the optimal power point. By doing this, the losses can be reduced by the difference between the line below the loss curve and the actual loss curve. The cost of switching on the fuel cell is disregarded and this mode of operation can be considered only if the

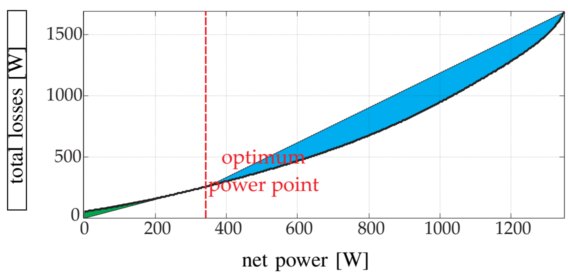

Fig. 4. Measured fuel cell losses as the function of the output power.

storage tank is large enough to switch only rarely.

2) Operation beyond optimal power point: Beyond the optimal power point, the losses cannot be reduced by alternating between different operating points. This is because the losses curve is both convex and always above the line crossing the origin. Because of this, the optimum is to provide only the average power with the fuel cell. The maximum loss reduction that could be obtained for the worst load with an infinite storage unit by providing all the AC power with the storage unit is the difference between the line crossing the optimum power point and the maximum power point and the actual loss curve.

D. Roles and control objectives

To fulfill the objective of reducing the hydrogen consumption, while respecting constraints, different roles are assigned to the different controllers in the proposed hierarchical control structure. The main role of the supercapacitor module is to bring or keep the DC-voltage common bus voltage close to its reference by manipulating the supercapacitor current reference, while respecting its maximum admissible current constraint. To maintain the bus voltage at its reference, the controller will quickly adjust its power to match all powers that affect the DC voltage balance. If there is a persisting power imbalance the supercapacitor storage unit will activate its maximum or minimum voltage constraints. To avoid this, the main role of the fuel cell module is to keep the supercapacitor tank voltage close to its reference by manipulating the fuel cell current reference while respecting its ramp rate and maximum admissible current constraints. This controller will be designed such that the fuel cell current varies as little as possible but such that the worst case load will not activate the supercapacitor module constraints. The main role of the two low level current controllers are to track the references given by the bus and supercapacitor voltage controllers.

E. Time scales

The rapidity requirements are different depending on the role of each block. The fastest controllers are the current control loops, which need to track the reference and deal with the fast time scale of the power converter filters. Their sampling time is the same T_{s1}. The second fastest controller is the DC bus voltage regulator, which needs to maintain the bus voltage despite load disturbance, which may be relatively abrupt. Its sampling time is denoted T_{s2}. The energy management control system needs to deal with the slow dynamics of the supercapacitor tank voltage and fuel cell ramp rate. It is the slowest loop and has a sampling time denoted T_{s3}.

F. Referential transformations

In the formulations that follow, we need to combine powers from different sources or sinks. It is however more convenient to manipulate currents to preserve the linearity of the manipulated dynamics. The currents need to be mapped from one referential to another considering the converter is a DC transformer. Assuming lossless conversion, the sum of currents is zero seen from all referentials

$$i^r_{\mathrm{sc},k} + i^r_{\mathrm{fc},k} - i^r_{\ell,k} \tag{1}$$

The transformation from one referential x to another r is obtained applying power conservation

$$\begin{gathered} i^r_{x,k} = \frac{v_{x,k}}{v_{r,k}} i^x_{r,k} \\ r, x \in \{\mathrm{sc}, \mathrm{fc}, \mathrm{b}\} \quad \mathrm{i_{b,k}} \equiv \mathrm{i}_{\ell,\mathrm{k}} \end{gathered} \tag{2}$$

where sc designate the supercapacitors, fc the fuel cell, b the bus and ℓ the load, while superscripts are employed to determine in which referential the current is seen. To simplify notations, we will omit r when $x = r$.

III. Supercapacitor and fuel cell MPC

The system to control comprises the supercapacitor storage unit and the fuel cell. The supercapacitor storage unit needs to be charged, while the fuel cell serves as charging device.

A. Fuel cell system dynamics

The fuel cell internal dynamics can be neglected at this level, only taking into account their limited dynamic capabilities by incorporating ramp constraints in the MPC formulation. We therefore only considers the supercapacitor tank dynamics resulting in the following discrete-time model

$$v_{\mathrm{sc},k+1} = v_{\mathrm{sc},k} + \frac{T_{\mathrm{s3}}}{C_{\mathrm{sc}}} \left(i^{\mathrm{sc}}_{\mathrm{fc},k} - i^{\mathrm{sc}}_{\ell,k} \right) \;, \tag{3a}$$

where $v_{\mathrm{sc},k}$ is the voltage of the supercapacitor tank, C_{sc} its capacitance. The key to obtain a linear model consists in bringing all the currents in the supercapacitor storage unit reference frame. The manipulated variable is therefore the current of the fuel cell viewed from the supercapacitor tank $i^{\mathrm{sc}}_{\mathrm{fc},k}$. The actual load current is computed as

$$i^{\mathrm{sc}}_{\ell,k} = \frac{v_{\mathrm{b},k}}{v_{\mathrm{sc},k}} i_{\ell,k} \;, \tag{3b}$$

where $v_{\mathrm{b},k}$ is the common DC link voltage, $v_{\mathrm{sc},k}$ the ultracapacitor tank voltage.

B. System constraints

The system constraints are the supercapacitor storage unit maximum and minimum voltages

$$v_{\mathrm{sc,min}} \le v_{\mathrm{sc},k} \le v_{\mathrm{sc,max}} \;, \tag{4a}$$

the fuel cell maximum admissible current

$$i_{\mathrm{fc,min}} \le i_{\mathrm{fc},k} \le i_{\mathrm{fc,max}} \;, \tag{4b}$$

and the fuel cell admissible current ramp rate constraints

$$-\Delta i_{\mathrm{fc,max}} \le i_{\mathrm{fc},k} - i_{\mathrm{fc},k-1} \le \Delta i_{\mathrm{fc,max}} \;. \tag{4c}$$

The two later constraints needs to be reformulated in the supercapacitor reference system to constrain the manipulated input

$$\frac{v_{\mathrm{fc},k}}{x_{\mathrm{sc},k}} i_{\mathrm{fc,min}} \le i^{\mathrm{sc}}_{\mathrm{fc},k} \le \frac{v_{\mathrm{fc},k}}{x_{\mathrm{sc},k}} i_{\mathrm{fc,max}} \tag{4d}$$

$$-\frac{v_{\mathrm{fc},k}}{v_{\mathrm{sc},k}}\,\Delta i_{\mathrm{fc,max}} \leq i^{\mathrm{sc}}_{\mathrm{fc},k} - i^{\mathrm{sc}}_{\mathrm{fc},k-1} \leq \frac{v_{\mathrm{fc},k}}{v_{\mathrm{sc},k}}\,\Delta i_{\mathrm{fc,max}} \tag{4e}$$

The reformulation in (4d) and (4e) results in nonlinear constraints. If the horizon is short, the fuel cell voltage and supercapacitor voltage are assumed constant, which makes the constraints linear. If the horizon is longer, then the constraints are linearized around the operating point at the beginning of the horizon, which makes them linear time varying.

C. MPC for direct loss minimization

The objective of minimizing the hydrogen losses can be written directly in function of the manipulated variables, assuming $v_{\mathrm{fc},l}\, i_{\mathrm{fc},l} = v_{\mathrm{sc},l}\, i^{\mathrm{sc}}_{\mathrm{fc},l}$

$$\min_{\boldsymbol{i}^{\mathrm{sc}}_{\mathrm{fc}}} \sum_{l=0}^{N_{\mathrm{fc}}-1} \ell_{\mathrm{fc}}(v_{\mathrm{sc},l}\, i^{\mathrm{sc}}_{\mathrm{fc},l}) + \frac{R_{\mathrm{sc}}\, C_{\mathrm{sc}}}{T_{\mathrm{s3}}}\,(v_{\mathrm{sc},l+1} - v_{\mathrm{sc},l})^2 \tag{5a}$$

$$\text{s.t. } (3a), (4a), (4d), (4e)\ . \tag{5b}$$

The cost function (5a) measures the real losses. The first term corresponds to the curve depicted in Fig. 4. It is either convex or can be approximated (precisely) by a convex function. The second term lumps all the conduction losses of the supercapacitor system unit including the power converter. Switching losses haven't been included but their approximation could easily be included. Considering the cost function is convex and given that all constraints are linear, the associated optimization problem can in principle be solved quite efficiently. There are, however, a few issues:

- the load needs to be modelled accurately over a long horizon, either in a deterministic or in a probabilistic way,
- the horizon needs to be long to anticipate activation of supercapacitor constraints.

If the load can be modelled efficiently, approach (5) will give optimal performance. In the next section, we introduce an alternative formulation for the case where the load cannot be modelled in advance.

D. MPC with sub-optimal indirect loss minimization

In section II-C, we have seen that the consumption is minimized when the fuel cell provides only the average power. The problem (5) can therefore be reformulated as tracking the average load current with the fuel cell. The issue is to determine the average load current over a finite horizon and to ensure satisfaction of the supercapacitor storage units constraints. Ensuring these constraints introduce some conservatism that reduces the achievable performance.

$$\min_{\boldsymbol{i}^{\mathrm{sc}}_{\mathrm{fc}}} \sum_{l=0}^{N_{\mathrm{fc}}-1} q_{\mathrm{sc1}}\,(v_{\mathrm{sc},l+1} - v_{\mathrm{sc,ref},l})^2 + q_{\mathrm{sc2}}\,(\Delta v_{\mathrm{sc},l})^2 + \left(i^{\mathrm{sc}}_{\mathrm{fc},l} - \bar{i}^{\mathrm{sc}}_{\ell,l}\right)^2 \tag{6a}$$

$$\text{s.t. } (3a), (4d), (4e), (4a)\ , \tag{6b}$$

where $\bar{i}^{\mathrm{sc}}_{\ell,l}$ is the estimated average load, that needs to be evaluated over the horizon of interest. The first term measures the deviation of the supercapacitor tank to its reference value. The second term measures the supercapacitor tank current. The third term measures the deviation between the fuel-cell operating point and the load. From the optimal solution to (6), the fuel cell current reference is obtained applying the inverse transformation

$$i_{\mathrm{fc,ref},k} = \frac{v_{\mathrm{sc},k}}{v_{\mathrm{fc},k}}\, i^{\mathrm{sc}}_{\mathrm{fc,ref},k}\ . \tag{6c}$$

IV. Bus voltage MPC

The system to control is the common DC bus. The supercapacitor is the controlled current source (manipulated variable) that allows charging the DC bus.

A. Bus voltage system dynamics

The fuel cell and supercapacitor dynamics are neglected for the control of the DC bus voltage. The discrete-time dynamics for the bus voltage control are very similar to (3a)

$$v_{\mathrm{b},k+1} = v_{\mathrm{b},k} + \frac{T_{\mathrm{s2}}}{C_{\mathrm{b}}}\,\left(i^{\mathrm{b}}_{\mathrm{sc},k} - i_{\ell,k} + i^{\mathrm{b}}_{\mathrm{fc},k}\right) \tag{7a}$$

C_{b} is the bus capacitance, $v_{\mathrm{b},k}$ is the bus capacitor voltage to be controlled. $v_{\mathrm{sc},k}$ is the measured supercapacitor tank voltage. Due to the large capacitance, this voltage is controlled by the higher level energy management and is considered as a measured disturbance. $i_{\ell,k}$ is the unmeasured load current. Due to its slow dynamics, the fuel cell is considered as static from the point of view of the bus voltage control. A key to formulate the problem is therefore to consider the current injected in the DC-bus by the fuel cell converter system $i^{\mathrm{b}}_{\mathrm{fc},k}$ as a known uncontrolled exogenous variable.

B. System constraints

The only constraint is the maximum supercapacitor tank current constraint

$$-i_{\mathrm{sc,max}} \leq i_{\mathrm{sc},k} \leq i_{\mathrm{sc,max}}\ . \tag{8a}$$

As before the constraint needs to be reformulated in the bus reference frame

$$-i_{\mathrm{sc,max},k}\,\frac{v_{\mathrm{sc},k}}{x_{\mathrm{b},k}} \leq i^{\mathrm{b}}_{\mathrm{sc},k} \leq i_{\mathrm{sc,max},k}\,\frac{v_{\mathrm{sc},k}}{v_{\mathrm{b},k}}\ . \tag{8b}$$

C. MPC problem formulation

$$\min \sum_{l=0}^{N_{\mathrm{b}}-1} q_{\mathrm{b1}}\,(v_{\mathrm{b},l+1} - v_{\mathrm{b,ref},0})^2 + \left(i^{\mathrm{b}}_{\mathrm{sc},l}\right)^2 \tag{9a}$$

$$\text{s.t. } (7a), (8b)\ . \tag{9b}$$

The cost criterion (9a) minimizes the bus voltage error while keeping the supercapacitor current small. The tradeoff between the two objectives is tuned by adjusting q_{b1}. The supercapacitor reference current is obtained by transforming the solution to (9) in the supercapacitor reference frame.

$$i_{\mathrm{sc,ref},k} = \frac{v_{\mathrm{b},k}}{v_{\mathrm{sc},k}}\, i^{\mathrm{b}}_{\mathrm{sc},k}\ . \tag{10}$$

V. BUCK CONVERTER MPC

The same scheme is applied for the control of the currents of both buck converters.

A. Buck converter system dynamics

The control model depicted in Fig. 5 is employed both for the fuel cell current controller and for the supercapacitor storage unit current controller. In this model, the fuel cell volt-

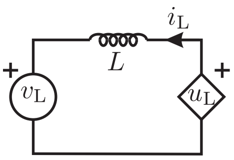

Fig. 5. Low level current control model

age v_{fc} (respectively supercapacitor voltage v_{sc}) and DC-bus voltages are considered as measured or estimated disturbances.

$$i_{\mathrm{L},k+1} = i_{\mathrm{L},k} + \frac{T_{\mathrm{s1}}}{L_{\mathrm{L}}} \left(u_{\mathrm{L},k} - v_{\mathrm{L},k}\right) \tag{11}$$

B. Buck converter system constraints

The switching leg is modeled employing averaging. It follows that the manipulated (averaged) voltage u is a continuous variable that must be bigger than 0 and smaller than the bus voltage v_{b}. The duty cycle is obtained by applying

$$d_k = \frac{u_k}{v_{\mathrm{b},k}} \quad v_{\mathrm{b},k} > 0 \quad 0 \le u_k \le v_{\mathrm{b},k} \tag{12}$$

This model has linear dynamics and constraints.

We have the following identities for the fuel cell current controller

$$i_{\mathrm{L}} \equiv i_{\mathrm{fc}} \quad v_{\mathrm{L}} \equiv v_{\mathrm{fc}} \quad L \equiv L_{\mathrm{fc}} \tag{13a}$$

and for the supercapacitor current controller

$$i_{\mathrm{L}} \equiv i_{\mathrm{sc}} \quad v_{\mathrm{L}} \equiv v_{\mathrm{sc}} \quad L \equiv L_{\mathrm{sc}}. \tag{14a}$$

C. MPC problem formulation

The MPC problem for the tracking of the current in a coil can be formulated very simply

$$\min_{\boldsymbol{u}_{\mathrm{L}}} \sum_{l=0}^{N_{\mathrm{L}}-1} q_{\mathrm{L1}} \left(i_{\mathrm{L},l+1} - i_{\mathrm{L,ref},0}\right)^2 + \left(u_{\mathrm{L},l} - v_{\mathrm{L},0}\right)^2 \tag{15a}$$

$$\text{s.t. } (11), (12) \ . \tag{15b}$$

VI. MPC IMPLEMENTATION ALGORITHMS

A. Explicit MPC

Optimization problems (6), (9) and (15) need to be solved at relatively high sampling frequencies, in the order of a few kHz. Since they are time invariant linear MPC problems or relatively small dimension, they can be solved parametrically off-line [17] and implemented as a look-up table on-line, which has the form of a binary search-tree [18]. The algorithm is written as follows

$$\begin{aligned} &\text{find } i \text{ s.t. } \boldsymbol{x} \in \mathcal{R}_i \ , \\ &u^{\mathrm{opt}} = \boldsymbol{K}_i \boldsymbol{x} + u_i \ , \end{aligned} \tag{16}$$

where $\boldsymbol{K}_i$, u_i and $\mathcal{R}_i$ have been computed off-line and where $\mathcal{R}_i$ is represented efficiently as a binary search-tree. $\boldsymbol{x}$ is the vector of parameters comprising estimated state and reference.

B. Online MPC

Due to time varying constraints, a long horizon and a convex cost that is neither linear nor quadratic, optimization(5a) cannot be solved offline using explicit MPC. It needs to be solved online, however this is realistic since the sampling period is much slower. (6) can moreover be employed to quickly get a suboptimal initial feasible solution.

VII. SIMULATION RESULTS

A. Current controllers

On the top plots of Fig. 6 and Fig. 7, we can see the response of the current MPC. We can verify that the reference in dash is very quickly and accurately tracked.

B. DC bus voltage controller

On the bottom plot of Fig. 6, we can see the response of the bus voltage controller. The DC bus voltage is regulated as quickly as possible by activating the current constraint (40 A).

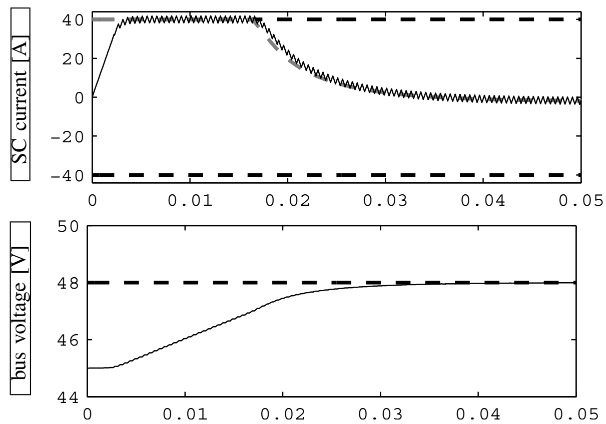

Fig. 6. Simulation results: response of supercapacitor storage unit current MPC (top) and bus voltage MPC (bottom)

C. Supercapacitor voltage controller

On the bottom plot of Fig. 7, we can see the response of the supercapacitor voltage controller. We can see on the top plot that the ramp constraints are respected. The supercapacitor unit is charged.

VIII. EXPERIMENTAL RESULTS

A. Experimental setup used to validate the MPC approach

The proposed innovative control laws have been tested on a small-scale experimental rig based on (see Fig. 8)

- a PEM fuel cell, Nexa 1200 W, 46 A, and 26 V, designed by Ballard. The selected PEM technology is the most mature candidate for portable and transportation applications (quick start-up and high power densities).
- a Maxwell supercapacitor bank, 26 F, 30 V, and 50 A designed by Maxwell Technologies

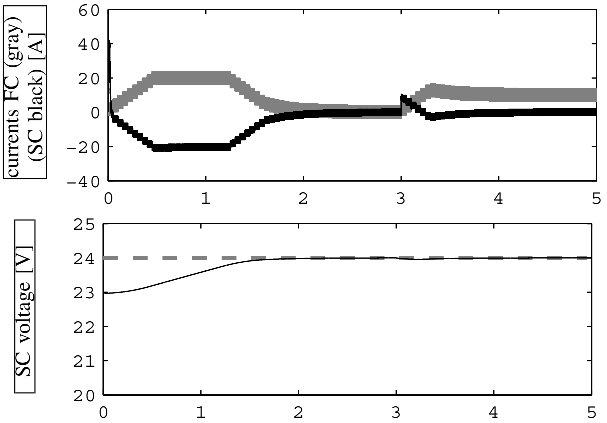

Fig. 7. Simulation results: response of fuel cell current MPC (top) and supercapacitor voltage MPC (bottom)

- a programmable electronic load,
- and a dSPACE DS1103 controller board. The control laws were implemented using Matlab/Simulink software.

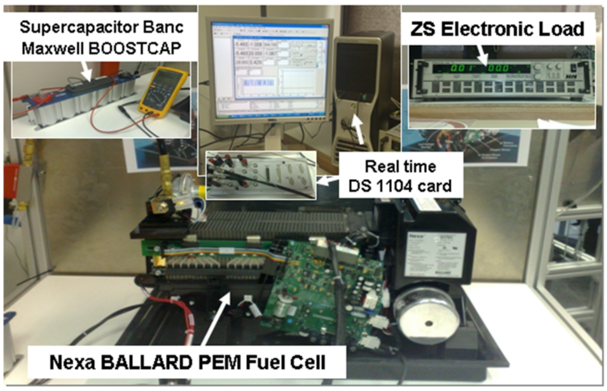

Fig. 8. Picture of the experimental rig with its main elements.

B. Experimental results

Fig. 9 illustrates some tests. We can verify that the supercapacitor is following abrupt profiles while the fuel cell only follows the load very slowly.

IX. Conclusion

A new hierarchical model predictive control scheme has been proposed for the control of hybrid systems with fuel cells and storage units. Three of the proposed MPC loops can be computed off-line parametrically and implemented very quickly as a binary searchtree lookup table. It is shown that there are two different modes of operation separated by the optimal power point to reduce the hydrogen consumption. Below optimal power point, the lowest consumption is obtained by operate only at optimum power point and zero power, turning off the fuel cell, provided the available storage is sufficient to prevent over-switching. Above the optimal power point the lowest consumption is obtained by operating as close as possible to average power, minimizing the deviations with the

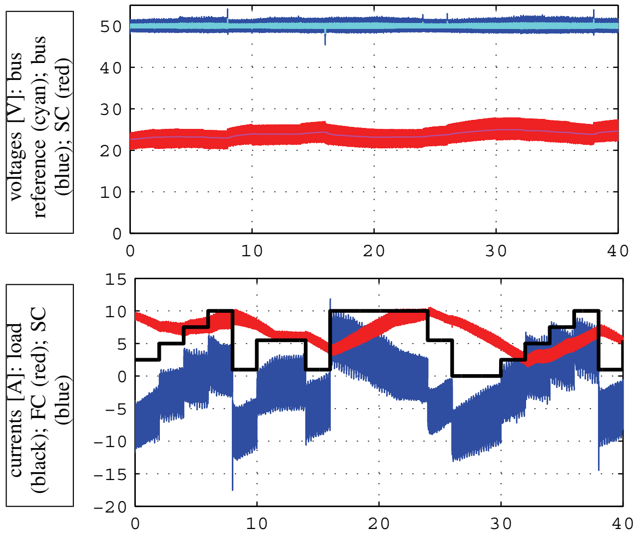

Fig. 9. Experimental results: application of a load profile (time in s)

supercapacitor storage unit. The proposed control concepts are verified both in simulation and experimentally.

References

[1] OECD/IEA. World energy outlook 2008. ed. by IEA Pub., Paris, 2008.

[2] OECD/IEA. Energy technology analysis, prospects for hydrogen and fuel cells. International Energy Agency, Dec. 2005.

[3] Frano Barbir. *Pem Fuel Cells: Theory And Practice*. Elsevier Academic Press, June 2005.

[4] R.K. Nema A. Kirubakaran, S. Jain. A review on fuel cell technologies and power electronic interface. *Renewable and Sustainable Energy Reviews*, 13(9):2430–2440, Dec. 2009.

[5] Young-Jun Sohn et al. Operating characteristics of an air-cooling pemfc for portable applications. *Journal of Power Sources*, 145(2), 2005.

[6] B. Wahdame et al. Comparison between two pem fuel cell durability tests performed at constant current under solicitations linked to transport mission profile. *Int. Journal of Hydrogen Energy*, 33(14), July 2008.

[7] M. Gerard et al. Oxygen starvation analysis during air feeding faults in pemfc. *Int. Journal of Hydrogen Energy*, 35(22), Nov. 2010.

[8] R. Trigui, B. Jeanneret, B. Malaquin, and C. Plasse. Performance comparison of three storage systems for mild hevs using phil simulation. *IEEE Trans. on Vehicular Technology*, 58(8):3959–3969, Oct. 2009.

[9] P. Thounthong, S. Rael, and B. Davat. Analysis of supercapacitor as second source based on fuel cell power generation. *IEEE Trans. on Energy Conversion*, 24(1):247 – 255, Jan. 2009.

[10] T. Azib, O. Bethoux, G. Remy, and C. Marchand. Saturation management of a controlled fuel-cell/ultracapacitor hybrid vehicle. *IEEE Trans. on Vehicular Technology*, 60(9):4127–4138, Nov. 2011.

[11] J. Golbert and D.R. Lewin. Model-based control of fuel cells: (1) regulatory control. *Journal of Power Sources*, 135(1–2):135 – 151, 2004.

[12] J. Golbert and D.R. Lewin. Model-based control of fuel cells: (2) optimal efficiency. *Journal of Power Sources*, 173(1):298 – 309, 2007.

[13] M.A. Danzer, S.J. Wittmann, and E.P. Hofer. Prevention of fuel cell starvation by model predictive control of pressure, excess ratio, and current. *Journal of Power Sources*, 190(1):86 – 91, 2009.

[14] J.K. Gruber et al. Nonlinear MPC for the airflow in a PEM fuel cell using a volterra series model. *Control Engineering Practice*, 20(2), 2012.

[15] A. Arce, A.J. del Real, and C. Bordons. MPC for battery/fuel cell hybrid vehicles including fuel cell dynamics and battery performance improvement. *Journal of Process Control*, 19(8):1289 – 1304, 2009.

[16] W. Greenwell and A. Vahidi. Predictive control of voltage and current in a fuel cell-ultracapacitor hybrid. *IEEE Trans. on Ind. El.*, 57(6), 2010.

[17] A. Bemporad, F. Borrelli, and M. Morari. Model Predictive Control Based on Linear Programming - The Explicit Solution. *IEEE Trans. on Automatic Control*, 47(12):1974–1985, December 2002.

[18] P. Tøndel, T.A. Johansen, and A. Bemporad. Evaluation of piecewise affine control via binary search tree. *Automatica*, 39(5):945–950, 2003.