

Complex photonic nonlinear delay dynamics for high performance signal and information processing

Nonlinear delay dynamics for information processing

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Abstract—Delay dynamics are known as infinite dimensional. They exhibit a wide variety of behavior, from stable steady states to chaos, through periodic solutions. Photonic appeared during the last decades as a fascinating physical playground, to both explore this complexity and also use it in signal and information processing applications.

Index Terms—Nonlinear dynamics, delay equations, photonic delay oscillator, complexity, information processing

I. UBIQUITOUS DELAY DYNAMICS, AND OPTICS

Although it is rarely taught in any academic program, delay dynamics can indeed be found in many real life situations [1]. They emerge from any physical, chemical, biological, etc... situation, where “long” range transport phenomena are present, and are coupled to dynamical phenomena (see Fig.1). Such transport mechanisms play the role of a memory for traveling temporal fluctuations. The fluctuations are stored within the transport channel, until they exit and they interact again with their source through a feedback mechanism, after having experienced dynamical processes and/or nonlinear transformations. The channel can be blood vessels, heat conduction medium, axons between neurons, and also wires or optical fibers.

The “length” of the transportation medium, or equivalently the delay τ_D it produces from the channel input to its output, has to be compared with the elementary duration (τ) allowed by the limiting dynamics (the differential process).

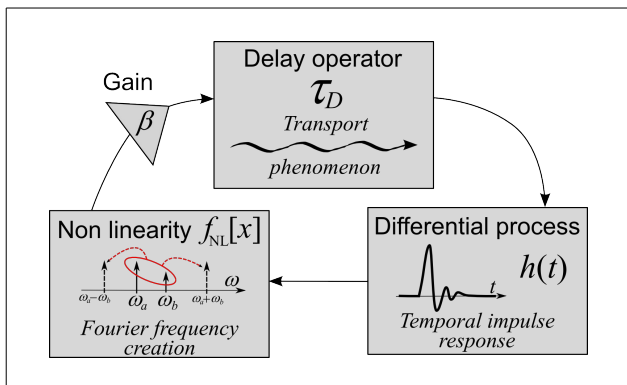


Fig. 1. Signal processing viewpoint of a nonlinear DDE oscillator. Delay is coupled to a linear filter and a nonlinear transformation within a feedback loop. A tunable gain selects the motion complexity.

This comparison or scaling (τ_D/τ) is a measure of the dimensionality of the complex dynamics that can be generated by the delay system. The long delay configuration is the one of interest when complex motions are desired.

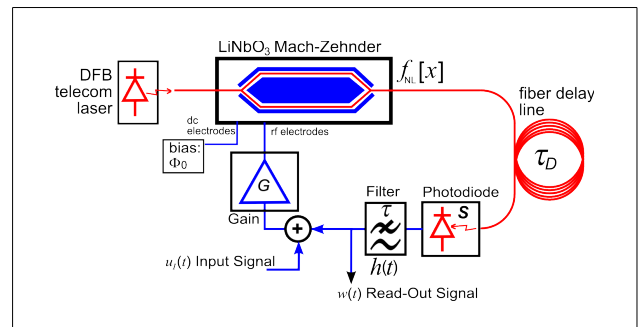


Fig. 2. Optoelectronic realization of a DDE oscillator, involving an integrated optic Mach-Zehnder modulator. Signal injection into the dynamics, as well as output monitoring is included, as usually of concern in signal processing application.

Photonics offers paradigmatic experiments [2] to explore and use the huge complexity potential of delay dynamics. Two such photonic delay systems are particularly popular in the literature, and they have been intensively studied for their numerous and rich dynamical behavior, among which one can obtain high dimensional optical chaos. The external cavity laser diode (ECLD) involves a Fabry-Pérot semiconductor laser subjected to a feedback of one output beam from an distant mirror [3]. The Ikeda ring cavity involves a localized Kerr medium inside a long ring cavity seeded by a CW laser beam. The results we report on, are concerned by the second example, however implemented physically with an electro-optic phase change [4] instead of the Kerr one, as illustrated in Fig.2. This experimental detail has two main consequences: The dynamics is scaled down to lower speed (however still allowing for the ones of typical optoelectronics telecom bandwidth); The parameter space is however also scaled up to wider ranges enabling experimental access to more complex dynamics. In this situation, the nonlinear transformation $f_{NL}[x]$ is provided by a two wave interference phenomenon, resulting in a \cos^2 -profile, which operating point (parameter Φ_0 in Fig.1) can be adjusted through a bias voltage for the electro-optic effect.

II. MODELING WITH SIGNAL PROCESSING VIEWPOINT

In the case of the Ikeda approach, a very convenient feature is to have nicely separated processes, as illustrated in Fig.1: The dynamical process is a limited rate of change modeled by a linear differential process in time, corresponding to a filtering transfer function $H(\omega)=\text{FT}[h(t)]$ in the Fourier space ($h(t)$ being the temporal impulse response of the filter, and FT the Fourier Transform operator); The nonlinear transformation $f_{\text{NL}}[x]$ performed by a two wave interference phenomenon (supposed to be adiabatic, i.e. instantaneous, without any dynamical limitation); And the time delayed feedback operation $x_{\tau_D}(t)=x(t-\tau_D)$, e.g. obtained through light propagation inside the ring cavity or a any light propagation path of a given length (as a fiber length for the optoelectronic approach). Modeling such a dynamics can be written mathematically in different forms [4]:

$$x(t) = [h * f_{\text{NL}}(x_{\tau_D})](t) = \int_{-\infty}^{+\infty} h(t-\theta) f_{\text{NL}}[x_{\tau_D}(\theta)] d\theta \quad (1)$$

$$X(\omega) = \text{FT}[x(t)] = H(\omega) \cdot \text{FT}\{f_{\text{NL}}[x(t)]\} \cdot e^{-i\omega\tau_D} \quad (2)$$

In the case of a first order linear differential process with a characteristic response time τ , $h(t)$ is the standard exponential decaying function $e^{-t/\tau}$ (for $t>0$, whereas $h(t)=0$ elsewhere), and one obtains the usual scalar delay differential equation (DDE):

$$x(t) + \tau (dx)/(dt) = f_{\text{NL}}[x_{\tau_D}(t)], \quad (3)$$

which is the case of the well-known Mackey-Glass [6] and Ikeda dynamics [7].

In the case of a damped oscillator differential process (damping factor m , central oscillating frequency $\omega_0=2\pi F_0$), an integro-differential delay equation (iDDE) is found:

$$(\omega_0/2m) \int_{t_0}^t x(\theta) d\theta + x(t) + (2m/\omega_0)(dx)/(dt) = f_{\text{NL}}[x_{\tau_D}(t)], \quad (4)$$

where two different situations can be typically encountered, the broadband case for strong damping ($m \gg 1$), and the narrow band case for small damping ($m \ll 1$). In standard nonlinear dynamic analysis, the integro-differential form is often converted into a set of two coupled first order differential equations [5], introducing an additional variable y , e.g. $(dy)/(dt) = x$.

In the previous modeling, one has to analyze the evolution dynamics with respect to a bifurcation parameter. This parameter is usually the weight of the nonlinear delayed feedback term. It is referred to as β in the previous equations, and it corresponds to a multiplicative factor in the nonlinear function, as illustrated in Fig.1. In the physical example of Fig.2, β is adjusted through the electronic gain G , or even more conveniently through the optical power level of the CW laser seeding the electro-optic Mach-Zehnder modulator.

III. BIFURCATIONS AND BEHAVIORS

When $\beta=0$, the system is operating in open loop, without any nonlinear delayed feedback, and its behavior is simply the one of the filter represented by its impulse response $h(t)$. For stable filters, a stable steady is obtained at the output. As β is increased, the autonomous system in Fig.1 can develop more and more complex solutions, from the stable steady state, to periodic oscillations, experiencing sometimes the well known period doubling route to chaos. Depending on the properties of the filter $H(\omega)$, other even more exotic motions can be obtained [5], such as chaotic breather, slow periodic stable solution, and even virtual Chimera states [8].

The motion complexity in DDE is both issued from the delay, and from the strength of the nonlinear transformation (parameter β). Because of the delay, the phase space dimension is infinite, since the size of the initial conditions required to uniquely define a solution, is not a finite set of values, but an infinite set of x -values corresponding to a functional $x(t)$ for t in $[-\tau_D, 0]$. This represents the filling of the system memory with a particular waveform on a bounded time interval spanning over one delay.

In chaotic regime, the chaotic attractor of an Ikeda DDE is known to be scaled by the quantity $\beta \tau_D / \tau$. This quantity represents the strength of the nonlinear creation of new frequencies (β), and the number of shortest time scale allowed by the filter bandwidth ($1/\tau_D$), which can fill the delay time interval.

IV. APPLICATIONS FOR DELAY DYNAMICS SOLUTIONS

A. Optical Chaos Communications

The discover of chaos in optical systems through Ikeda setups or ECLD triggered a lot of attention. When the area of secure communication was proposed as an application of chaos, thanks to the synchronization capability of these motions despite their sensitivity to initial conditions, photonic was rapidly considered as an attractive area where it could be developed. Fiber optical communication could indeed implement in hardware this concept, thus offering physical layer encryption directly operating at the high speed of these communication systems. This was resulting in strong speed related difficulties for any eavesdropper willing to attack the links with computer-based means.

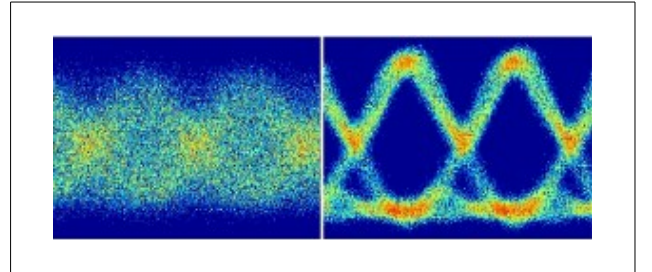


Fig. 3. 10Gb/s Eye diagrams of experimental chaos communication demonstration. Left: blurred data by optical chaos. Right: decoded data after chaos replication at the receiver side.

Optical chaos communication was then intensively addressed by many groups worldwide. The European consortium of the OCCULT project succeeded first in a field experiment, reaching Gb/s real time encryption and decryption [9], with both an ECLD setup, and an electro-optic intensity setup very similar to the scheme in Fig.2. Later, our group pushed this record further to 10Gb/s for a single channel, using a novel electro-optical phase chaos generator [10].

B. High spectral purity microwave oscillator

From a completely opposite perspective, instead of the white noise like motion corresponding to the chaotic optical waveform generated by a strongly nonlinear ($\beta \gg 5$) delay oscillator, the periodic solution of the same electro-optic delay architecture was also considered as an attractive new oscillator topology. These so-called OEOs (optoelectronic oscillators, [11]) were found to have, under specific parameter operating conditions, extremely low phase noise, and thus extremely high spectral purity in the Fourier domain (thus opposed to the flat white noise signature of the chaotic motion). Typical applications targeted by these microwave oscillators (from a few GHz to several 10s of GHz) are high resolution Radars, where the frequency multiplied quartz oscillators (10-100 times 100MHz) can not provide such very low phase noise features.

The typical operating conditions of OEOs, from the modeling point of view reported here, are concerned by extremely long delay lines (few km, instead of a few 10s of cm to meters for chaos communication), and very low damping m so that a single delay mode operation can be obtained in the desired microwave range. Although the dimensionality of the dynamics can be expected to be reduced due to the very small bandwidth of the oscillator (since $m \ll 1$), the fact that a large delay line is used counterbalances the bandwidth reduction by an also strongly reduced delay mode spacing.

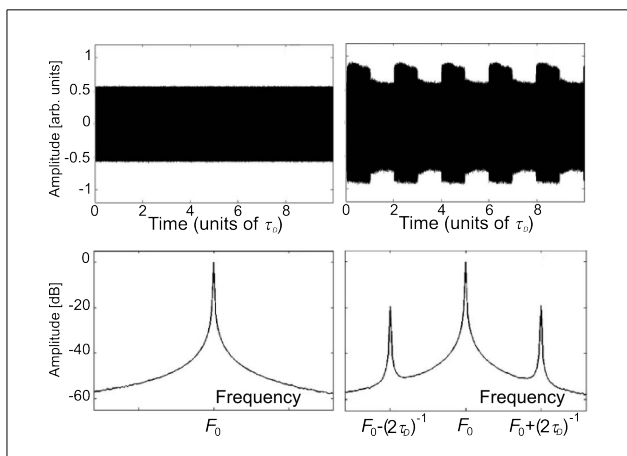


Fig. 4. Time (upper) and Fourier (lower, FFT) domain showing the OEO microwave oscillation before (left) and after (right) its destabilization due to an increased enough feedback gain.

The quantity $\beta \tau_D / \tau$ can thus still be potentially high for increased β : one should expect the high spectral purity microwave to be destabilized for some sufficiently large β , which is indeed what our group could demonstrate theoretically, numerically, and experimentally [12], as illustrated in Fig.4.

C. Brain-inspired photonic computing

Last, but hopefully not least, we could address a third application concerned by another asymptotic solution of a photonic delay dynamics. This asymptotic solution is surprisingly the most simple one, the single stable steady state (for small β ; notice bistability of fixed points is here most probably undesired). Its main attractive feature is related to the fact that it provides a generic property of any stable filter, the fading memory: after any perturbation, the transient dynamics necessarily goes back to this steady state, providing a reference initial condition for a later perturbation. Perturbation has to be viewed here as an input information signal $u_i(t)$, as represented in the setup of Fig. 2. This input signal triggers a transient from the stable steady state, which role is to embed the input information into the high-dimensional phase space of the nonlinear delay dynamic (high ratios for τ_D / τ are thus required). A machine learning approach is then involved to extract from this nonlinear transient, through a linear combination of points in the phase space, some hidden information which is expected to be contained in the input signal. This new computational concept is usually referred as to “Reservoir Computing” (originally named Echo State Network, and also Liquid State Machine), and is originating from the neural network computing and brain research communities.

Within a recent EU project PHOCUS, we demonstrated the first Photonic Reservoir Computer [13,14], with the architecture as depicted in Fig.2, where the linear Read-Out was extracted from the output signal of the delay dynamics.

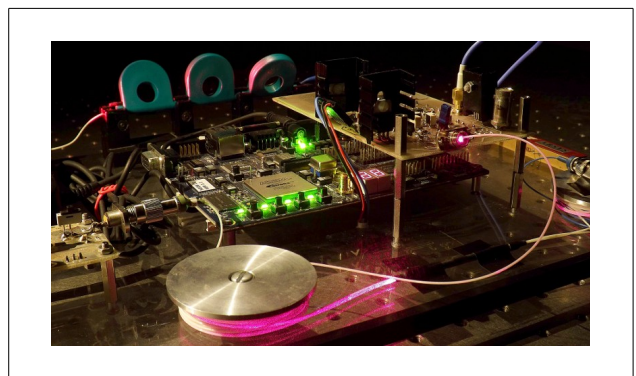


Fig. 5. Photonic Reservoir Computer, implemented with an FPGA (Field Programmable Gate Array) card for the post-processing Read-Out.

Spoken digit recognition, as well as time series prediction were successfully demonstrated, moreover with performances close to, or even beyond, the state of the art.

V. CONCLUSION

We have reported on three original applications, cryptography, high spectral purity microwave source for Radars, and brain-inspired computational approach. The understanding of the underlying complexity and high-dimensional nonlinear dynamics features allowed to design real-world photonic setups with which the feasibility of breakthrough concepts have been demonstrated. We anticipate that delay dynamics complexity, and its investigation with reliable and efficient photonic setups, will offer even more perspectives of other advanced applications, as it can be illustrated by very recent fundamental results on delay based dynamics, such as Chimera states in delay dynamics [8], and also modeling of WGM (whispering gallery modes) seeded by a laser light [15].

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