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A NEW NUMERICAL OPTIMIZATION APPROACH FOR STANDING-WAVE THERMOACOUSTIC ENGINES

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This paper develops a novel method to optimize standing-wave thermoacoustic engines with a parallel-plate stack using the particle swarm optimization method. The aim of the present work is to understand the effect of geometric, thermal and pressure parameters on the performance of a thermoacoustic engine. In particular, the studied parameters include: the resonators' length and diameter, stacks' length, hydraulic radius, porosity and position in the resonator, hot and cold temperature, frequency, mean pressure and drive ratio. To attain this objective, the Particle Swarm Optimization (PSO) method is highlighted and used in order to optimize this high number of parameters in one thermoacoustic problem which is, for the best of our knowledge, investigated for the first time in the literature. In this paper, the linear theory is applied to calculate the acoustics' pressure and velocity of a numerical model which consists of three sections, hot resonator, stack and cold resonator sections. Both the exergetic efficiency and the acoustic power produced are the two objective functions to be optimized. Results show that when exergetic efficiency is high the acoustic power produced is low and vice-versa. So, a third function that combines the two functions is optimized in order to have acceptable and meaningful values of both exergetic efficiency and acoustic power produced. Finally, significant results, which are useful to design any new thermoacoustic devices, are showed and discussed.

1. Introduction

Rayleigh was first explained qualitatively the thermoacoustic phenomenon in 1878 “If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged”[1]. A century later, Rott[2] gave the first concrete theory and quantitatively accurate understanding of thermoacoustic phenomenon. Since that, the interest in thermoacoustic systems has arisen and expanded. Benefiting from Rott’s approximation on thermoacoustic problems, Swift from Los Alamos National Laboratory and Garret from PEN State University have successfully led the field by making the first devices that produce a useful acoustic work[3,4].

A thermoacoustic system represents many advantages. It can use any external energy sources, it has no or few moving parts, it is friendly environmental and it is a low cost machine. Thus, to build a thermoacoustic device, it is sufficient to have a porous medium, stack or regenerator, sandwiched between hot and cold heat exchangers inside a resonator. In spite of all these assets, the exergetic efficiency of a thermoacoustic device is still relatively low and need to be improved by keeping, at the same time, an acceptable and meaningful value of the acoustic power produced by the device. To reach this objective, an optimization of a thermoacoustic system is highly recommended. However, thermoacoustic problem has no closed form solution and there are a lot of parameters that could affect the performance of a thermoacoustic device. Therefore, the classical optimization methods cannot be easily used in this kind of problems unless in some limited case such as some authors recently did in the literature[5–7]. But instead, there is a way more efficient to optimize a complex problem such as thermoacoustic problems. It is about the Particle Swarm Optimization method (PSO) which is a type of evolutionary computations and intelligence optimizations.

After we have obtained an encouraging result by applying the PSO method on a simple thermoacoustic problem[8], a more complicated thermoacoustic model is represented and studied in this paper. It consists of calculating the evolution of the acoustic pressure, velocity and temperature, by using the Rott's thermoacoustic approximation, for a numerical two closed end parallel plate stack thermoacoustic engine model. This model consists of three sections, a hot temperature zone, a parallel plate stack zone and a cold temperature zone. Therefore, the acoustic work produced by the stack, the exergetic efficiency and the product of both acoustic power and exergetic efficiency of the stack are optimized. The last function is chosen to be studied because results show that when exergetic efficiency is high the acoustic power produced is low and vice-versa. So, this third function grants acceptable and meaningful values of both exergetic efficiency and acoustic power produced. The performance of these three functions are studied as a function of different parameters at the same time: resonator's length and diameter, stacks' length, hydraulic radius, porosity and position in the resonator, hot and cold temperature, frequency, mean pressure and drive ratio. Some significant results regarding the optimal of each optimized function are derived and discussed.

2. Introduction to the Particle Swarm Optimization method

PSO is an iterative method that tries to maximize or minimize a function or problem in a known search-space of dimension N. Every problem has multiple candidate solutions, called particles, which are characterized by their positions and velocities over mathematical formulae. The PSO was invented by Kennedy and Eberhart[9] in 1995, inspired by social behavior of bird flocking or fish schooling.

Suppose the function to be optimized, f, is defined as:

$$f\colon \begin{cases} \mathbb{R}^N & \rightarrow \mathbb{R} \\ \vec{x} & \rightarrow f(\vec{x}) \end{cases} \tag{1}$$

Where $\vec{x}$ is the position vector of one particle. Then, let P be the number of particles in the swarm, where each particle has a position vector of dimension N. So, the position vector of a particle i is defined as $\vec{x}_i$, where $i = 1, \dots, P$, and $\vec{v}_i$ is its velocity.

Let k be the number of iterations, then the algorithm for maximizing a function can be summarized as:

I- Initialize randomly the position vector, $\vec{x}_0^i$, and the velocity vector, $\vec{v}_0^i$ for each particle. Then, evaluate the function $f_0^i(\vec{x}_0^i)$. Therefore, update the velocity vector, $\vec{v}_1^i$, by using Eq. 2, and the position vector $\vec{x}_1^i = \vec{x}_0^i + \vec{v}_1^i$ for each particle.

II- For each iteration, $j = 1, \dots, k$

- For each particle, $i = 1, \dots, P$
  - Evaluate the fitness value of the function, $f_j^i(\vec{x}_j^i)$ at position $\vec{x}_j^i$,
  - If $f_j^i(\vec{x}_j^i) > f_{best}^i(\vec{x}_{best}^i)$, then $f_{best}^i(\vec{x}_{best}^i) = f_j^i(\vec{x}_j^i)$ and $\vec{x}_{best}^i = \vec{x}_j^i$, where $f_{best}^i(\vec{x}_{best}^i)$ is the best value retained by a particle i at position $\vec{x}_{best}^i$
  - If $f_j^i(\vec{x}_j^i) > f_{best}^g(\vec{x}_{best}^g)$, then $f_{best}^g(\vec{x}_{best}^g) = f_j^i(\vec{x}_j^i)$ and $\vec{x}_{best}^g = \vec{x}_j^i$, where $f_{best}^g(\vec{x}_{best}^g)$ is the global best value retained at position $\vec{x}_{best}^g$

- Update particle velocity, $\vec{v}_{j+1}^{i}$, by using Eq. 2
- Update particle position, $\vec{x}_{j+1}^{i}$, where $\vec{x}_{j+1}^{i} = \vec{x}_{j}^{i} + \vec{v}_{j+1}^{i}$
- If stopping conditions are satisfied, go to step III

III- Report results and terminate.

In this paper, the constriction method is used to calculate the update particle's velocity[10,11]:

$$\vec{v}_{j+1}^{i} = 0.729\vec{v}_{j}^{i} + 1.494\vec{U}_1(0,1) \times \left(\vec{x}_{best}^{i} - \vec{x}_{j}^{i}\right) + 1.494\vec{U}_2(0,1) \times \left(\vec{x}_{best}^{g} - \vec{x}_{j}^{i}\right) \tag{2}$$

Where, $\vec{U}_1(0,1)$ and $\vec{U}_2(0,1)$ are two random vectors in which each component goes from zero to one. The velocity $\vec{v}$ is limited to $[\vec{x}_{min}, \vec{x}_{max}]$. This constriction method has a high success rate[12], and hence, it decreases the risk of premature convergence to non-optimal points.

3. The numerical model of thermoacoustic engine

The model is consisted of three sections as shown by Fig.1:

- Hot temperature zone: T_h is the hot temperature, D is the resonator's diameter and l_h is the length of this section
- Parallel plate stack zone: ξ is the stack's porosity, l_s is the stack's length and r_h is the stack's hydraulic radius
- Cold temperature zone: T_c is the cold temperature, D is the resonator's diameter and l_c is the length of this section

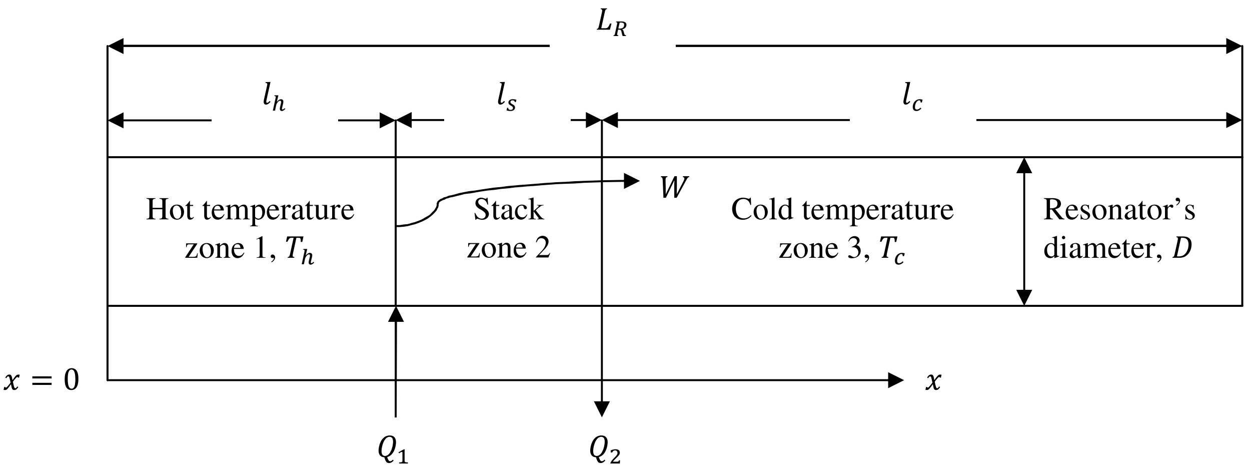

Figure 1. The thermoacoustic engine model

The rest of parameters that are used in this model are:
The mean pressure $\bar{p}_g$, the frequency fr, the drive ratio Dr which is the ratio between the acoustic pressure amplitude and the mean pressure, the heat added to the system Q_1, the heat ejected from the system Q_2, the acoustic power produced by the stack W which is equal to the difference between the stack's outgoing and incoming acoustic power, and finally there is the thermophysical properties of the gas used which is the Helium in our case.

Because the model has two closed end, the acoustic volumetric flow is equal to zero at the beginning and at the end of the model, $\bar{u}_a(x=0) = \bar{u}_a(x=L_R) = 0$. This also leads to have a maximum amplitude of the acoustic pressure at the model's start, $p_a(x=0) = Dr \times \bar{p}_g$, and a minimum amplitude at the model's end, $p_a(x=L_R) = -Dr \times \bar{p}_g$. As well, the cold temperature is supposed to be around 300K, $T_c \approx 300K$. Furthermore, the continuity of the acoustic pressure and volumetric flow and the temperature are assured in the intersections. In addition, the resonator's length is not supposed to exceed $L_R \leq 25\ m$. The drive ratio is supposed to be less than 7% to ensure that the model is working on a low-amplitude. In another word, a low-amplitude gives a good agreement between linear thermoacoustic equations and experiments. These are the boundary conditions values which are the keys used to solve the numerical model described below.

Besides, the system is supposed to be ideally insulated. Thus, there is neither external nor internal heat source exchanged with the system unless the heat added at the beginning of the stack $(at\ x = l_h)$ and the heat ejected from the system at the stack's end $(at\ x = l_h + l_s)$. Therefore, from the first law of thermodynamics, the total power of the system $Q_1 = Q_2 + W$.

3.1. Methodology

The calculation methodology is summarised in the following three steps:

I- The mean pressure $\bar{p}_g$, the frequency fr, the drive ratio Dr, the resonator's diameter D, the stack's porosity ξ, hydraulic radius r_h and length l_s, the temperature T_h and the length l_h of the hot temperature zone are fixed.

II- Based on the thermoacoustic equations, developed by Rott and written by Swift[13] (Eqs. 3-5), Q_1 and L_R are calculated numerically in order to satisfy the boundary conditions values explained before.

$$\frac{\partial p_a}{\partial x} = \frac{\bar{\rho}_g \omega}{i(1-g_\upsilon)*A} \bar{u}_a \tag{3}$$

$$\frac{d\bar{u}_a}{dx} = \frac{\omega * A}{i\gamma \bar{p}_g}[1 + (\gamma - 1)g_t]p_a + G_0 \frac{1}{\bar{T}_g} \left(\frac{Q_1 - \frac{1}{2}\Re\left[p_a \tilde{\bar{u}}_a \left(1 + \frac{\tilde{g}_\upsilon - g_t}{(1-\tilde{g}_\upsilon)(1+\mathrm{Pr})}\right)\right]}{\left(\frac{\bar{\rho}_g c_{pg}}{2\omega A(1-Pr^2)|1-g_\upsilon|^2}\left|\tilde{\bar{u}}_a\right|^2 \Im[g_t + Pr\tilde{g}_\upsilon] - A_s k_s - A k_g\right)} \right) \bar{u}_a \tag{4}$$

$$\frac{d\bar{T}_g}{dx} = \frac{Q_1 - \frac{1}{2}\Re\left[p_a \tilde{\bar{u}}_a \left(1 + \frac{\tilde{g}_\upsilon - g_t}{(1-\tilde{g}_\upsilon)(1+\mathrm{Pr})}\right)\right]}{\left(\frac{\bar{\rho}_g c_{pg}}{2\omega A(1-Pr^2)|1-g_\upsilon|^2}\left|\tilde{\bar{u}}_a\right|^2 \Im[g_t + Pr\tilde{g}_\upsilon] - A_s k_s - A k_g\right)} \tag{5}$$

Where, p_a and $\bar{u}_a$ are the acoustical oscillating pressure and volumetric flow, $\bar{T}_g$ is the gas mean temperature, $g_\upsilon = \frac{(1-i)\delta_\upsilon}{2r_h}$ and $g_t = \frac{(1-i)\delta_t}{2r_h}$ for sections 1 and 3 in the numerical model, $g_\upsilon = \frac{\tanh\left(\sqrt{2i}\frac{r_h}{\delta_\upsilon}\right)}{\sqrt{2i}\frac{r_h}{\delta_\upsilon}}$ and $g_t = \frac{\tanh\left(\sqrt{2i}\frac{r_h}{\delta_t}\right)}{\sqrt{2i}\frac{r_h}{\delta_t}}$ for section 2. $G_0 = \frac{(g_\upsilon - g_t)}{(Pr-1)(1-g_\upsilon)}$, δ_υ and δ_t are the viscous and thermal penetration depth, Pr is the Prandtl number, $\bar{\rho}_g$ is the mean density for an ideal gas, ω is the angular frequency, i is the imaginary unit, γ is isobaric to isochoric specific heat ratio, $\Re[\,]$ and $\Im[\,]$ are the real and imaginary parts, the tilde represents the conjugate number, k_s and k_g are the solid and gas thermal conductivity, A and A_s are the gas and solid area and c_{pg} is the gas isobaric heat capacity.

III- Once Q_1 and L_R are determined, the acoustic power produced by the stack and therefore the exergetic efficiency are calculated as:

$$W = W(x = l_h + l_s) - W(x = l_h) \tag{6}$$

$$\eta_{ex} = \frac{W}{Q_1}\frac{T_h}{T_h - T_c} \tag{7}$$

3.2. A numerical example and proof of the methodology

Steps I to III of section 3.1 are applied to a large number of random examples. The obtained results are very interesting and are all satisfied the boundary conditions values. Also, the same examples, taken randomly, were applied to the *Design Environment for Low-amplitude Thermoacous-*

tic Energy Conversion (DELTAEC)[14] software which is developed by Los Alamos National Laboratory and it is available for free. This software is widely used and trusted by thermoacousticians to design their thermoacoustic devices. The results obtained by our methodology and those obtained through DELTAEC show a high correspondence.

A comparison between our methodology and DELTAEC software for one of the random examples is shown here (refer to Table 1). The selected fixed parameters of the example are:
$\bar{p}_g = 10\ bar, fr = 50\ Hz, Dr = 0.05, D = 6\ cm, \xi = 0.8, r_h = 0.5\ mm, l_s = 0.2\ m, T_h = 700\ K$ and $l_h = 0.5\ m$.

Table 1. The difference between our methodology and DELTAEC for the taken example

parameters	Q_1	L_R	$\bar{u}_a(0)$	$\bar{u}_a(L_R)$	T_c	W	η_{ex}
Our methodology results	555.7 W	10.4 m	0 m^3/s	0 m^3/s	300.1 K	75.2 W	23.7%
DELTAEC results	555.7 W	10.4 m	0 m^3/s	0 m^3/s	300.2 K	75.2 W	23.7%
Error	0%	0%	0%	0%	0.03%	0%	0%

Moreover, the evolution of the acoustic pressure and volumetric flow and the gas mean temperature, for the selected example applied on both our methodology and DELTAEC, are shown in Fig.2. As a result, the error between our methodology and DELTAEC is almost zero. This demonstrates the efficiency of the methodology used in this paper. However, while DELTAEC software cannot be used to optimize a high number of parameters, our methodology, which is used jointly with PSO, can achieve this.

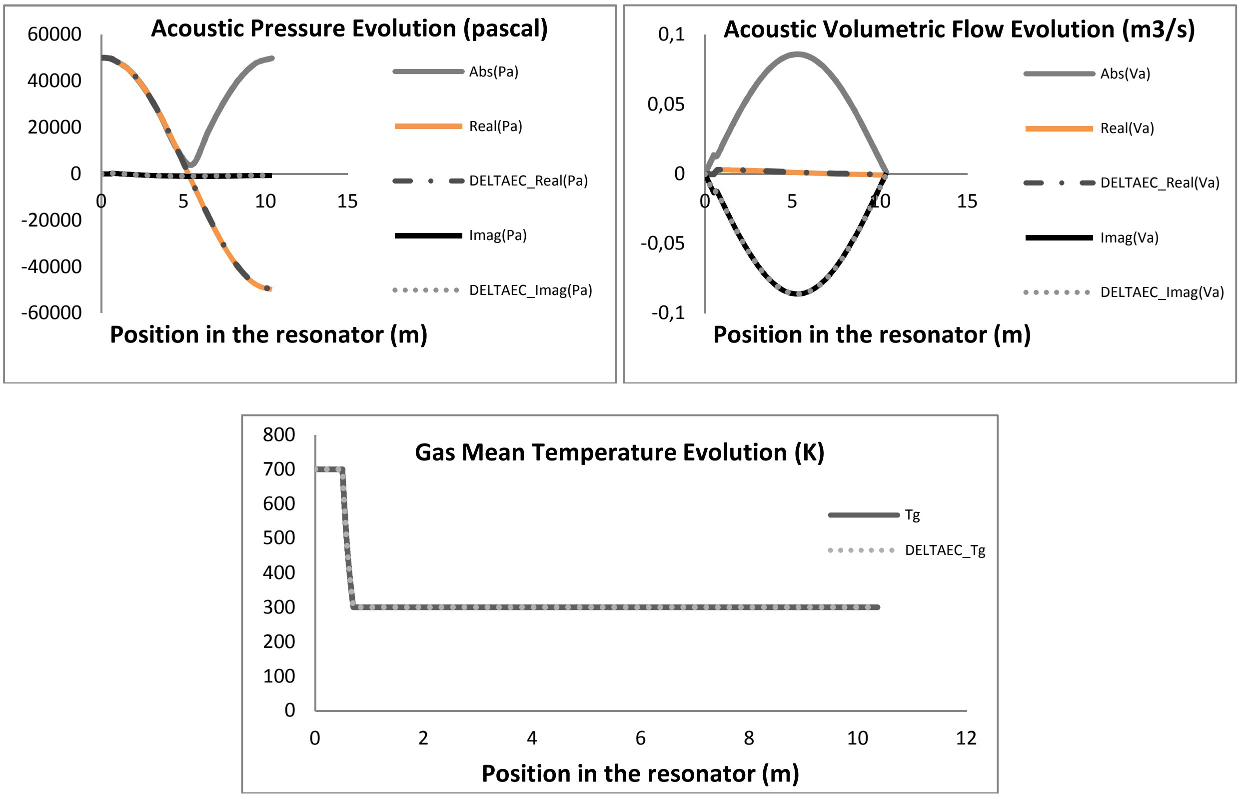

Figure 2. The evolution of the acoustic pressure and volumetric flow and the gas mean pressure for the selected example applied on both our methodology and DELTAEC

3.3. Numerical simulations

Once the methodology is explained and verified, the PSO method and therefore the algorithm of section 2 is applied to the methodology of section 3.1. The problem dimension, *N*, is equal to nine as can be deduced from step I of section 3.1. The search space of each dimension is summarized in Table 2. The number of particles is taken to be 24 and each simulation is performed 500 iterations. As a result, at each iteration, each particle does the steps I to III of section 3.1.

Table 2. The search space used for the PSO

$\bar{p}_g$	fr	D	r_h	l_s	T_h	l_h	ξ	Dr
Bar	Hz	cm	mm	m	K	m	%	%
[1;50]	[5,500]	[3;6]	[0.1;3]	[0.01;0.5]	[500;1000]	[0.01;12]	[5;95]	[0.7;7]

The PSO method is applied to maximize three functions, the acoustic power produced, the exergetic efficiency and the product of the first two functions.

3.4. Results and discussion

Fig.3 shows the evolution of the three optimized functions by the PSO method. Also, the results of the acoustic power, exergetic efficiency and the acoustic power times exergetic efficiency are represented respectively in Table 3, 4 and 5. As the PSO method is an iterative method, each function is run three times over a 500 iterations to ensure that the optimized value is converged to the global best value. From our experience, this number of iterations gives a good ratio between the calculation time and the convergence. We remind that the PSO initialization step is done randomly for each run.

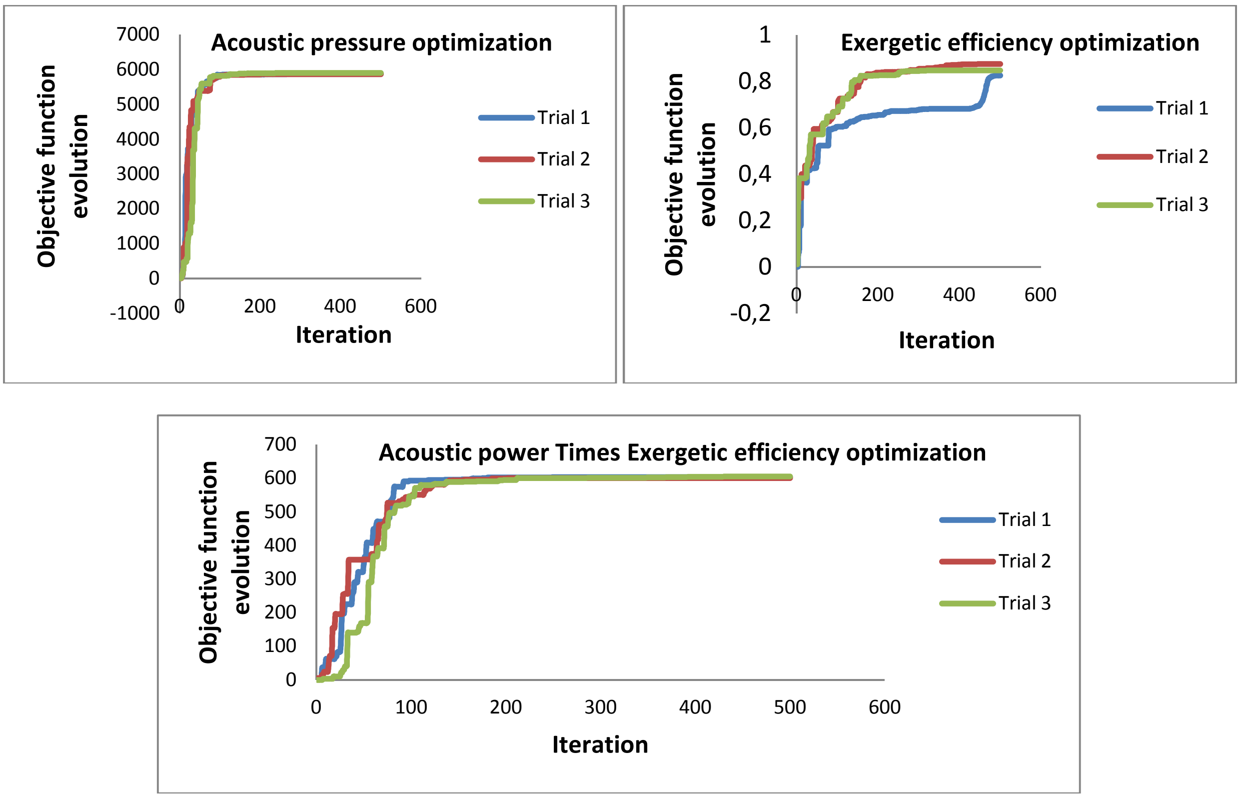

Figure 3. The PSO evolution of objective functions

When the acoustic power is maximized, the results show that the exergetic efficiency is very low. To attain the best value of the acoustic power, the following parameters should be at their maximum: the mean pressure, the resonator's diameter, the hot temperature, the stack's porosity and the drive ratio. In addition, the stack's length should be at its minimum, which is achieved by increasing the temperature gradient in the stack. The thermoacoustic engine should be worked at relatively low frequency, around 50 Hz. For the stack's position in the resonator, the results show that it should be around 40% of the resonator's length $l_h \approx 40\%\ of\ L_R$. Concerning the hydraulic radius, it should be around 0.4 mm where the mean thermal penetration depth inside the stack is around 0.3 mm.

Table 3. The acoustic power function optimization results

Trial	$\bar{p}_g$	fr	D	r_h	l_s	T_h	l_h	ξ	Dr	L_R	Q_1	W	η_{ex}
	bar	Hz	cm	mm	m	K	m	%	%	m	Mw	w	%
1	50	51	6	0.39	0.01	1000	4.5	95	7	11.5	6.5	5863	0.13
2	50	54	6	0.37	0.01	1000	4.3	95	7	10.8	6.5	5858	0.13
3	50	33	6	0.45	0.01	1000	7	95	7	17.5	9.4	5899	0.09

For the exergetic efficiency maximization, the resonator's diameter and hot temperature should be at their minimum while the stack's porosity and the drive ratio should be at their maximum. The stack's position in the resonator should be near the first closed end, $l_h \approx 0.15m$. Concerning the mean pressure, it would be around 25 bar and the engine should work at low frequency. In this paper, the lowest frequency available is around 20 Hz which corresponds to a resonator's length of 25 m. For the stack's length, it should be around 20 cm. The hydraulic radius in this case is around 0.25 mm while the mean thermal penetration depth inside the stack is equal to 0.41 mm. The acoustic power in this case is very low as shown the results.

Table 4. The exergetic efficiency function optimization results

Trial	$\bar{p}_g$	fr	D	r_h	l_s	T_h	l_h	ξ	Dr	L_R	Q_1	W	η_{ex}
	bar	Hz	cm	mm	m	K	m	%	%	m	w	w	%
1	24.7	20.2	3	0.3	0.24	500	0.17	95	7	25	2.9	1	87.4
2	47.2	20.2	3	0.18	0.19	500	0.15	95	7	25	7.2	2.4	82.4
3	46.4	20.3	3	0.21	0.20	500	0.14	95	7	25	4.4	1.5	84.7

As the best value of the acoustic power is corresponding to a very low value of the exergetic efficiency and vice-versa, the acoustic power times exergetic efficiency is optimized. This function gives an acceptable and meaningful value of both the acoustic power and the exergetic efficiency as the results shown in Table 5. To reach the maximum of this function, the following parameters should be kept at their maximum: the mean pressure, the resonator's diameter, the hot temperature, the stack's porosity and the drive ratio. The engine should be worked at relatively high frequency. A frequency of 125 Hz is a good approach for this example. For the stack's position in the resonator, it should be around $l_h \approx 14\%\ of\ L_R$. While the stack's length of 0.4 m gives a good approach. Finally the hydraulic radius of the stack should be around 0.22 mm which is near the mean penetration depth inside the stack, 0.23 mm.

Table 5. The acoustic power Times exergetic efficiency function optimization results

Trial	$\bar{p}_g$	fr	D	r_h	l_s	T_h	l_h	ξ	Dr	L_R	Q_1	W	η_{ex}
	bar	Hz	cm	mm	m	K	m	%	%	m	Kw	w	%
1	50	125.1	6	0.22	0.39	1000	0.59	95	7	4.2	12.9	2365	26
2	50	172.2	6	0.19	0.28	1000	0.43	95	7	3.1	12.8	2346	26
3	50	93.3	6	0.25	0.5	1000	0.81	95	7	5.6	13.8	2446	25

4. conclusions

A new numerical approach to optimize a thermoacoustic engine, by using the Particle Swarm Optimization (PSO) method, is represented in this paper. It consists of applying the PSO method into a simple numerical two closed end parallel plate stack thermoacoustic engine model. The model is consisted of three sections, a hot temperature, a stack and a cold temperature zone. Then, three functions depending on nine parameters are optimized. The functions optimized are the acoustic power, the exergetic efficiency and the product of both acoustic power and exergetic efficiency. The nine parameters are: the mean pressure, the frequency, the drive ratio, the resonator's diameter, the hot resonator's length, the hot temperature, the stack's porosity, hydraulic radius and length.

The results show that when the acoustic power is maximized, its exergetic efficiency is very low and vice-versa. So, it is not a good idea to design a thermoacoustic engine only based on the optimization of the acoustic power or the exergetic efficiency. That is why the acoustic power times the exergetic efficiency function is proposed to be optimized in this paper. This function gives a good approach of both the acoustic power and the exergetic efficiency.

The interesting part of this paper is, in addition to the use of the PSO method in thermoacoustic problems, the optimization of a complex thermoacoustic problem with a high number of parameters. The results were obtained within a few hours only instead of weeks in comparison with the classical optimization methods. Also, another advantage of using the PSO method is the possibility of optimizing a multi-objective function as the acoustic power times the exergetic efficiency function.

Regarding the interesting results shown in this paper, our future works will focus on modifying the numerical model represented in this paper by replacing one closed end by a mechanical charge, which could represent an electrical generator, and by taking into account the work lost in the resonator. Thus, future works will emphasize on the design of a thermoacoustic engine to generate electricity.

REFERENCES

1. Rayleigh L. The explanation of certain acoustical phenomena. *Nature*. 1878;18:319-321.
2. Rott N. Thermoacoustics. *Advances in Applied Mechanics*. 1980;20:135-175.
3. Swift GW. Thermoacoustic engines and refrigerators. *Physics Today*. 1995;48(7):22-28.
4. Garrett SL. Resource letter: Ta-l: thermoacoustic engines and refrigerators. *American Journal of Physics*. 2004;72(1):11-17.
5. Kang H, Li Q, Zhou G. optimizing hydraulic radius and acoustic field of the thermoacoustic engine. *Cryogenics*. 2009;49(3-4):112-119.
6. Hu ZJ, Li ZY, Li Q, Li Q. Evaluation of thermal efficiency and energy conversion of thermoacoustic Stirling engines. *Energy Conversion and Management*. 2010;51(4):802-812.
7. Babaei H, Siddiqui K. Design and optimization of thermoacoustic devices. *Energy Conversion and Management*. 2008;49:3585-3598.
8. Chaitou H, Nika P. Exergetic optimization of a thermoacoustic engine using the particle swarm optimization method. *ENERGY CONVERSION AND MANAGEMENT*. 2011;55:71-80.
9. Kennedy J, Eberhart R. Particle Swarm Optimization. *IEEE*. 1995;4:1942-1948.
10. Clerc M. The swarm and the queen: towards a deterministic and adaptive particle swarm optimization. *Proc. ICEC, Washington, DC*. 1999:1951-1957.
11. Eberhart R, Shi Y. Particle Swarm Optimizatio: Developments Applications and Resources. *IEEE*. 2001:81-86.
12. Trelea IC. The particle swarm optimization algorithm: convergence analysis and parameter selection. *Information Processing Letters*. 2003;85:317-325.
13. Swift G. *Thermoacoustics: A unifying perspective for some engines and refrigerators*. 5th ed. (LANL, ed.).; 2001.
14. Ward WC, Clark JP, Swift GW. Design Environment for Low-amplitude Thermoacoustic Energy Conversion (DELTAEC), Los Alamos National Laboratory: http://www.lanl.gov/thermoacoustics/DeltaEC.html.