Automatic solver for computational plasticity based on Taylor series and Automatic Differentiation

A. Lejeune*, H. Boudaoud†, N. Mathieu† and M. Potier-Ferry†

*Institut FEMTO-ST, Université de Franche-Comté, F-25000 Besancon, France,
email: arnaud.lejeune@univ-fcomte.fr

† LEM3, Université de Lorraine, F-57045 Metz, France,
emails: hakim.boudaoud@univ-lorraine.f, norman.mathieu@univ-lorraine.fr, michel.potier-ferry@univ-lorraine.fr

ABSTRACT

A numerical technique is presented to compute high order derivatives of response curves in plasticity, i.e. high order derivatives of plastic constitutive laws and high order derivatives of the corresponding structural problems. Of course the transitions between elastic unloading and plastic loading lead to discontinuous tangent moduli and it is necessary to regularize the constitutive laws to be able to compute Taylor series within plasticity. In [1], regularization methods have been proposed that do not affect the effectiveness of structural calculations and we adopt them in this talk.

Often plastic laws are rather intricate, many physical models can be considered and algorithms to compute Taylor series are not simple. Therefore the implementation of this procedure should be user friendly by using Automatic Differentiation (AD) [2]. That is the goal of this paper. AD could be viewed as a way to generate a code that computes the derivatives of a function, but only the function itself has to be implemented by the user. A natural approach is the so called “forward mode” based on operator overloading. In [3], the same idea had been applied to two algebraic laws: nonlinear elasticity and unilateral contact with tools of any form. Here a generic procedure for Differential Algebraic Equations is discussed and applied to small strain and large strain plasticity, but it could be naturally extended to friction.

The application to structural problems is straightforward because it is easy to calculate derivatives of balance laws. The combination of both leads to Asymptotic Numerical Method that has been applied to many cases, especially in nonlinear elasticity and in fluid mechanics [4]. Applications to the computations of plastic structural problems will be discussed.

REFERENCES

[1] M. Assidi, N. Damil, M. Potier-Ferry and H. Zahrouni, ”Regularization and perturbation techniques to solve plasticity problems”, *Int. J. Mat. Form.*, **2**, 1-14 (2009).

[2] A. Griewank, *Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation*, Frontiers in Appl. Math., vol. 19, SIAM (2000).

[3] A. Lejeune, H. Boudaoud, M. Potier-Ferry, I. Charpentier, H. Zahrouni,” Automatic solver for non-linear partial differential equations with implicit local laws. Application to unilateral contact” *Int. J. Num. Meth. Engng*, to appear.

[4] B. Cochelin, N. Damil and M. Potier-Ferry, *Méthode Asymptotique Numérique*, Hermes (2007).