# Steady-state and instabilities of octave-spanning Kerr frequency combs modeled using a generalized Lugiato-Lefever equation

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Since first demonstrated in 2007, the generation of frequency combs in microresonators has attracted significant interest. A small footprint, high power efficiency and on-chip potential make them attractive substitutes for commercially existing comb sources [1]. Extensive experimental research on such Kerr combs has followed, but theoretical analyses are comparatively scarce. This deficiency can be linked to the intractable computational complexity of the existing models [2]. Here we report that octave-spanning Kerr frequency combs can be realistically and efficiently modeled with a generalized Lugiato-Lefever (LL) equation [3], which has been extensively used in the past to model macroscopic optical fiber resonators. Steady-state Kerr comb solutions in good agreement with reported experiments can be obtained by solving this equation with a Newton-Rhapson root-finding algorithm, a method which is orders-of-magnitude faster than any other technique, even when including more modes than ever before. Additionally, using split-step Fourier integration of the LL model, dynamical instabilities of Kerr frequency combs can be studied in a similarly fast fashion. Our results reveal characteristic spectral signatures of such instabilities.

We consider numerical parameters approximating the experiment of Ref. [4], i.e., a 200 μm-diameter critically-coupled SiN resonator with a loaded Q of $3{\cdot}10^5$, a nonlinear coefficient $\gamma = 1$/W/km, and coupled pump powers within 400–800 mW. Looking for a steady-state comb solution of the LL model for this configuration, we obtain the octave-spanning spectrum shown in Fig. 1(a). The agreement with experiment is relatively good (see Fig. 2 in [4]). Looking at a time-frequency representation [Fig. 1(b)], we see that the simulated intracavity electric field consists of a *single* pulse atop a weak continuous-wave background. This structure can be readily identified as a temporal cavity soliton (CS), which is the well known dissipative soliton of nonlinear passive resonators [5]. Moreover, we can see how such soliton emits a dispersive component at 2150 nm, directly related to the corresponding narrowband feature in Fig. 1(a). We interpret this component as a resonant dispersive wave (DW) emitted by the CS underlying the frequency comb, and indeed the wavelength of this DW is correctly predicted by known phase-matching conditions [dashed horizontal line in Fig. 1(b)]. For slightly different coupling conditions, however, split-step Fourier simulations reveal unstable solutions characterized by a turbulent evolution of *multiple* CSs simultaneously present in the resonator. Figs 1(c) and (d) illustrate *averaged* spectra of such solutions for an increasing level of turbulence. In fact, we note that the spectrum shown in Fig. 1(d) is in even better agreement with the experiment of Ref. [4] than Fig. 1(a). This suggests that the peaks symmetrically located around the pump and the broadened DW component are characteristics of instabilities and that some observed spectra may only appear stable because of the averaging of spectrum analyzers.

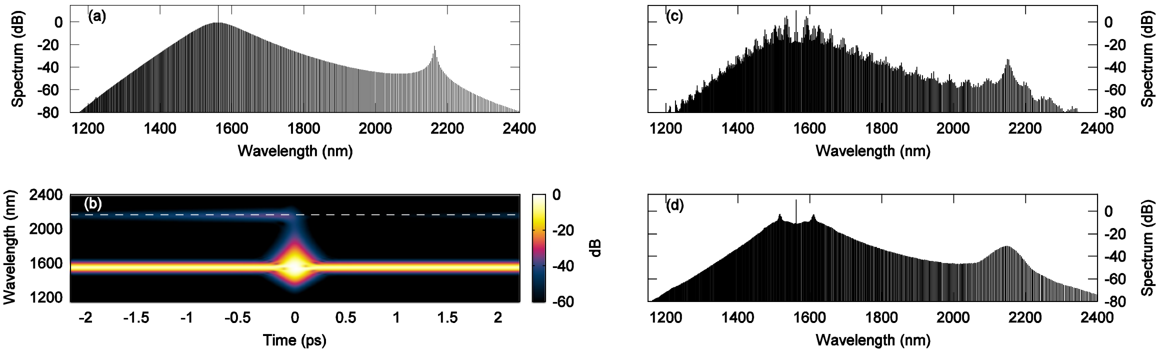

**Fig. 1** (a) Steady-state Kerr comb solution of the LL equation with parameters from [4]. (b) Corresponding time-frequency representation. The horizontal line is the predicted DW wavelength. (c) and (d) are *averaged* spectra of unstable solutions involving multiple CSs per roundtrip.

## References

[1] T. J. Kippenberg, R. Holzwarth, and S. A. Diddams, "Microresonator-Based Optical Frequency Combs," Science **332,** 555-559 (2011).
[2] Y. K. Chembo and N. Yu, "Modal expansion approach to optical-frequency-comb generation with monolithic whispering-gallery-mode resonators," Phys. Rev. A **82,** 033801/1–18 (2010).
[3] S. Coen, H. G. Randle, T. Sylvestre, and M. Erkintalo, "Modeling of octave-spanning Kerr frequency combs using a generalized mean-field Lugiato-Lefever model," Opt. Lett. **38,** 37–39 (2013).
[4] Y. Okawachi, K. Saha, J. S. Levy, Y. H. Wen, M. Lipson, and A. L. Gaeta, "Octave-spanning frequency comb generation in a silicon nitride chip," Opt. Lett. **36,** 3398–3400 (2011).
[5] F. Leo, S. Coen, P. Kockaert, S.-P. Gorza, P. Emplit, and M. Haelterman, "Temporal cavity solitons in one-dimensional Kerr media as bits in an all-optical buffer," Nat. Photon. **4,** 471–476 (2010).