International Conference Mathematics Days in Sofia
July 7–10, 2014, Sofia, Bulgaria

Grothendieck's dessins d'enfants in quantum contextuality

Michel Planat

Institut FEMTO-ST, CNRS, 32 Avenue de l'Observatoire, 25044 Besançon, France
michel.planat@femto-st.fr

Keywords: quantum contextuality, Grothendieck's dessins d'enfants, permutation groups, finite geometries, geometric hyperplanes.

In quantum mechanics, due to the non-commutativity of observables, the reality of the particle prior to the measurement has to be questioned. This reality relies on the arrangement set up and on compatible unperformed/counterfactual measurements: this is called quantum contextuality (QC). Proofs of QC, the Kochen-Specker theorems [1], rely on remarkable point/line incidence geometries $\mathcal{G}$'s of the projective (or polar type), as the Tits generalized polygons [2].

In this work, one finds that such finite geometries arise from (are stabilized by) some Grothendieck's dessins d'enfants [3]. A dessin d'enfant is a bipartite graph embedded in a Riemann surface defined over the field $\bar{\mathbb{Q}}$ of algebraic numbers (i.e. an algebraic curve). Following Grothendieck's "Esquisse d'un programme", the (two-generator) permutation group P of a $\mathcal{D}$, of prescribed characteristics, is readily derived from the cosets of the relevant subgroup of the free group on two generators. Then, the two-point stabilizers of P ensure the derivation of the corresponding $\mathcal{G}$'s.

Restricting to $\mathcal{G}$'s with three points on a line, we also investigate the organization of their geometric hyperplanes h_i, whose (set theoretical) 'addition law' $h_i \oplus h_j$ is the complement of their symmetric difference.

Acknowledgements. Part of this work is currently performed jointly with the authors of [2].

References

[1] M. Planat, On small proofs of the Bell-Kochen-Specker theorem for two, three and four qubits *EPJ Plus* **127**, 86 (2012).

[2] M. Planat, A. Giorgetti, F. Holweck and M. Saniga, Quantum contextual finite geometries from *dessins d'enfants*, Preprint 1310.4267 [quant-ph].

[3] S. K. Lando and A. K. Zvonkin, Graphs on surfaces and their applications (Springer Verlag, 2004).