Electrimac 2011
University of Cergy-Pontoise

Special Session 4 :
Analytical Models in Electromagnetic Devices

June 6-8th, 2011, Paris, France

Slotting Effect in Permanent-Magnet Motors via a 2-D Exact Sub-Domain Model

F. Dubas and **C. Espanet**

femto-st sciences & technologies

Outlines

- Context and Objective of the Work
- Review of the Existing Analytical Models
- 2-D Analytical Field Model (i.e., Sub-Domains)
- Full Quantities No-Load Calculation
- Conclusion

Introduction: Context and Objective

- **Accurate knowledge of the magnetic field distribution in the air-gap.**
- **Multi-pole surface mounted permanent-magnet (PM) motors (SMPMM).**
- **Modeling of slotting effect.**
- **Full quantities no-load evaluation.**
- **New 2-D analytical Solution.**
- **Comparison analytical model vs Finite Element Analysis (FEA 2-D)**

Review of the Existing Analytical Models

Electromagnetic Modeling

- **Numerical Method**
- **Analytical Method**
- **Permeance Network**

Maxwell's equations

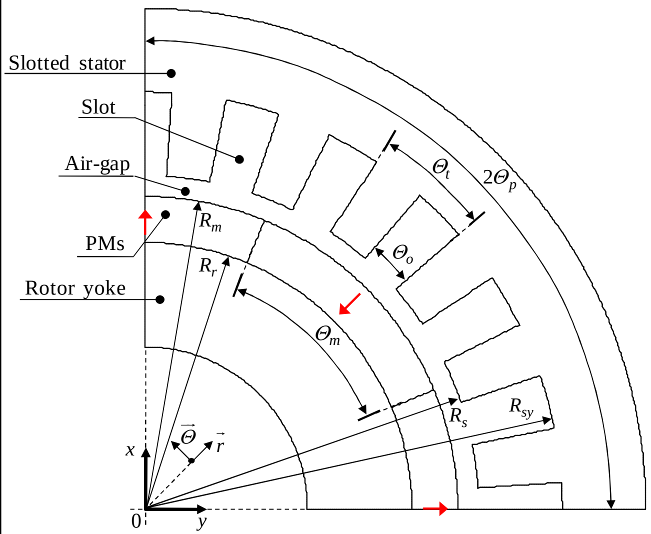
- **1-D Analytical Model.**
- **2-D Slotless Analytical Model [1].**
- **Slotting Effect:**
  1. **Conformal Transformation (Relative or Complex Permeance Model);**
  2. **Exact Sub-Domain [2].**

[1] F. Dubas, "Conception d'un Moteur Rapide à Aimants Permanents pour l'entraînement de Compresseurs de Piles à Combustible", Ph.D Dissertation, UFC, Belfort, France, 2006.

[2] F. Dubas and C. Espanet, "Analytical Solution of the Magnetic Field in Permanent-Magnet Motors Taking Into Account Slotting Effect: No-Load Vector Potential and Flux Density Calculation", IEEE Trans. on Magn., vol. 45, no. 5, pp. 2097-2109, May 2009.

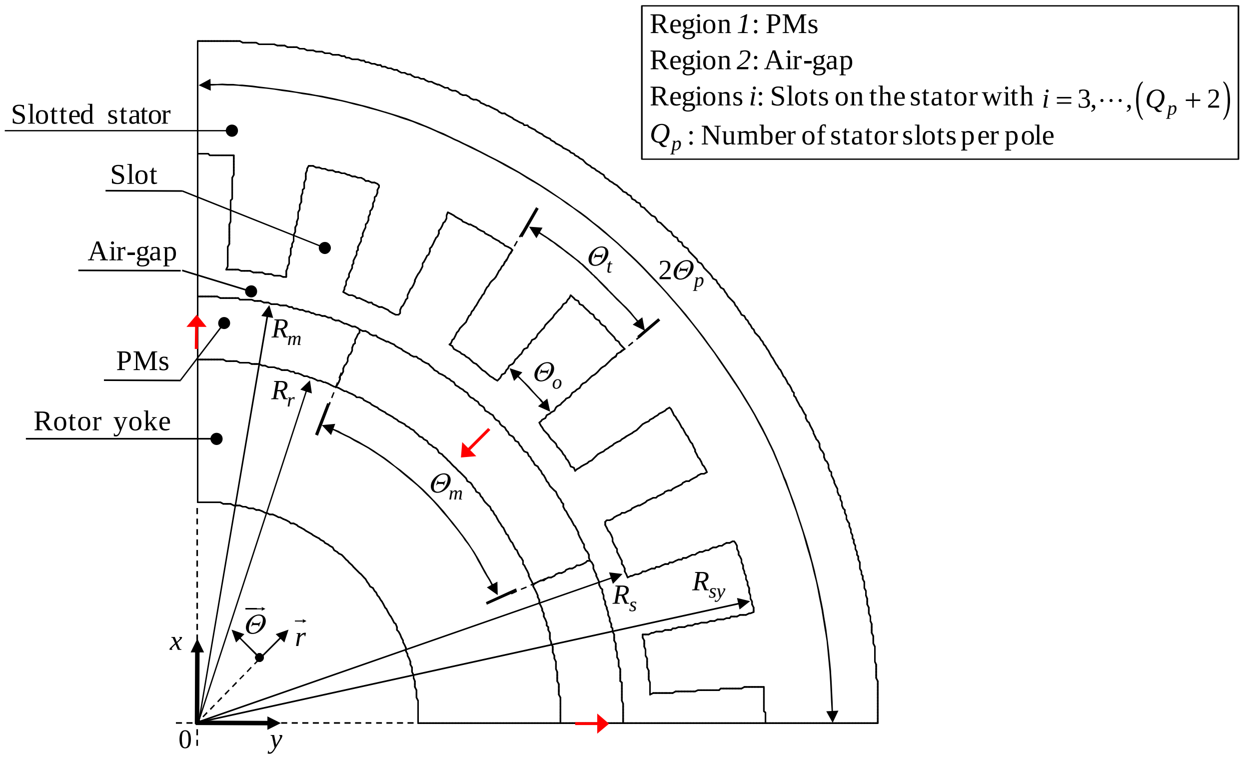
Problem Description and Assumptions

Cross section under a one pole-pair of the multi-pole SMPMM



- **End-effects are neglected.**
- **Saturation is neglected.**
- **Non-conductive PMs material (i.e., non-resolution of Diffusion's equations).**
- **Radial/Parallel magnetization.**
- **Linear demagnetization characteristics of PMs.**
- **Radial slot faces on the stator.**

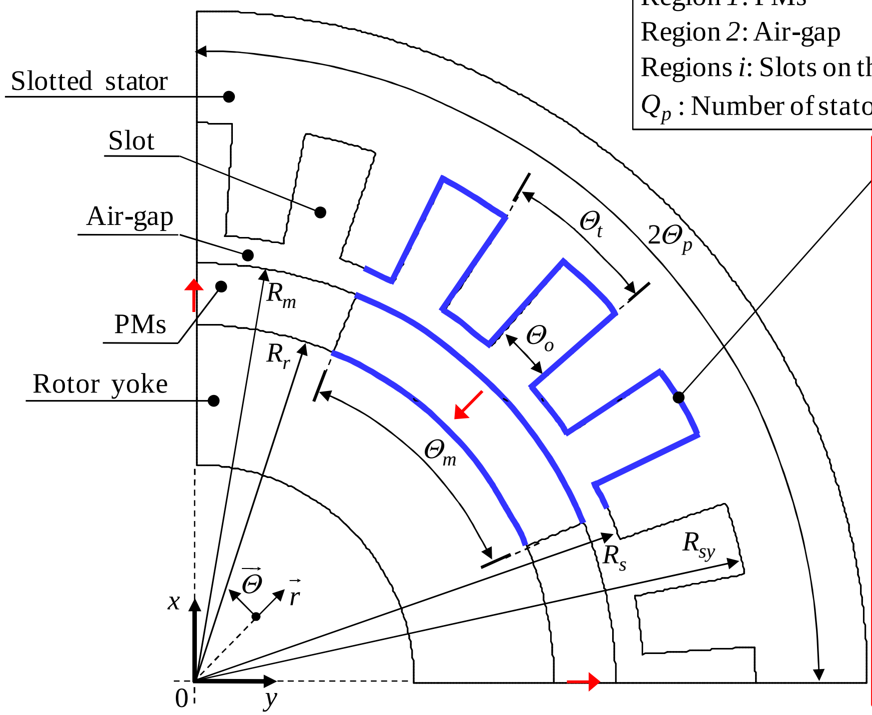
2-D Exact Sub-Domain Model



- **Poisson's equations**: $\Delta A_{z1} = B_{rm} \cdot K_m(r, \Theta_s)$ **in the PMs.**
- **Laplace's equations**: $\Delta A_{z2} = 0$ **in the air-gap.**

  $\Delta A_{zi} = 0$ **in the slots on the stator.**

2-D Exact Sub-Domain Model



Region 1: PMs
Region 2: Air-gap
Regions i: Slots on the stator with $i = 3,\cdots,(Q_p+2)$
Q_p : Number of stator slots per pole

Boundary conditions between PMs/Air-gap/ Slots on the stator

- $r = R_r$: $H_{\Theta 1} = 0 \quad \forall \Theta_s$
- $r = R_m$: $B_{r2} = B_{r1}$, $H_{\Theta 2} = H_{\Theta 1}$ $\quad \forall \Theta_s$
- $r = R_s$: $B_{ri} = B_{r2}$, $H_{\Theta i} = H_{\Theta 2}$ $\quad -\frac{\Theta_o}{2} + \Theta_i \le \Theta_s \le \frac{\Theta_o}{2} + \Theta_i$
  otherwise $H_{\Theta 2} = 0$
- $r = R_{sy}$: $H_{\Theta i} = 0 \quad -\frac{\Theta_o}{2} + \Theta_i \le \Theta_s \le \frac{\Theta_o}{2} + \Theta_i$
- $\Theta_s = \pm \frac{\Theta_o}{2} + \Theta_i$: $B_{ri} = 0 \quad R_s \le r \le R_{sy}$

- **In the PMs:** $A_{z1} = B_{rm} \cdot R_m \cdot f_{z1}(E_{1n}, G_{1n}, r, \Theta_s)$
- **In the air-gap:** $A_{z2} = B_{rm} \cdot R_m \cdot f_{z2}(E_{2n} \sim H_{2n}, r, \Theta_s)$
- **In the slots on the stator:** $A_{zi} = B_{rm} \cdot R_s \cdot f_{zi}(F_{iv}, r, \Theta_s)$

2-D Exact Sub-Domain Model

❑ The linear Cramer's system:

$$[IC] = [Q]^{-1} \cdot [K]$$

Integration Constants → $[IC]$

Boundary conditions matrix → $[Q]$

Magnetic sources (i.e., **PM and Current**) → $[K]$

❑ Size of the Cramer's system, i.e., $[Q]$:

$6 \cdot Nn + Q_p \cdot Nv$ **equations and unknowns.**

with Nn and Nv are the number of terms in the Fourier's series for the computation A_{z1} , A_{z2} and A_{zi}

2-D Exact Sub-Domain Model

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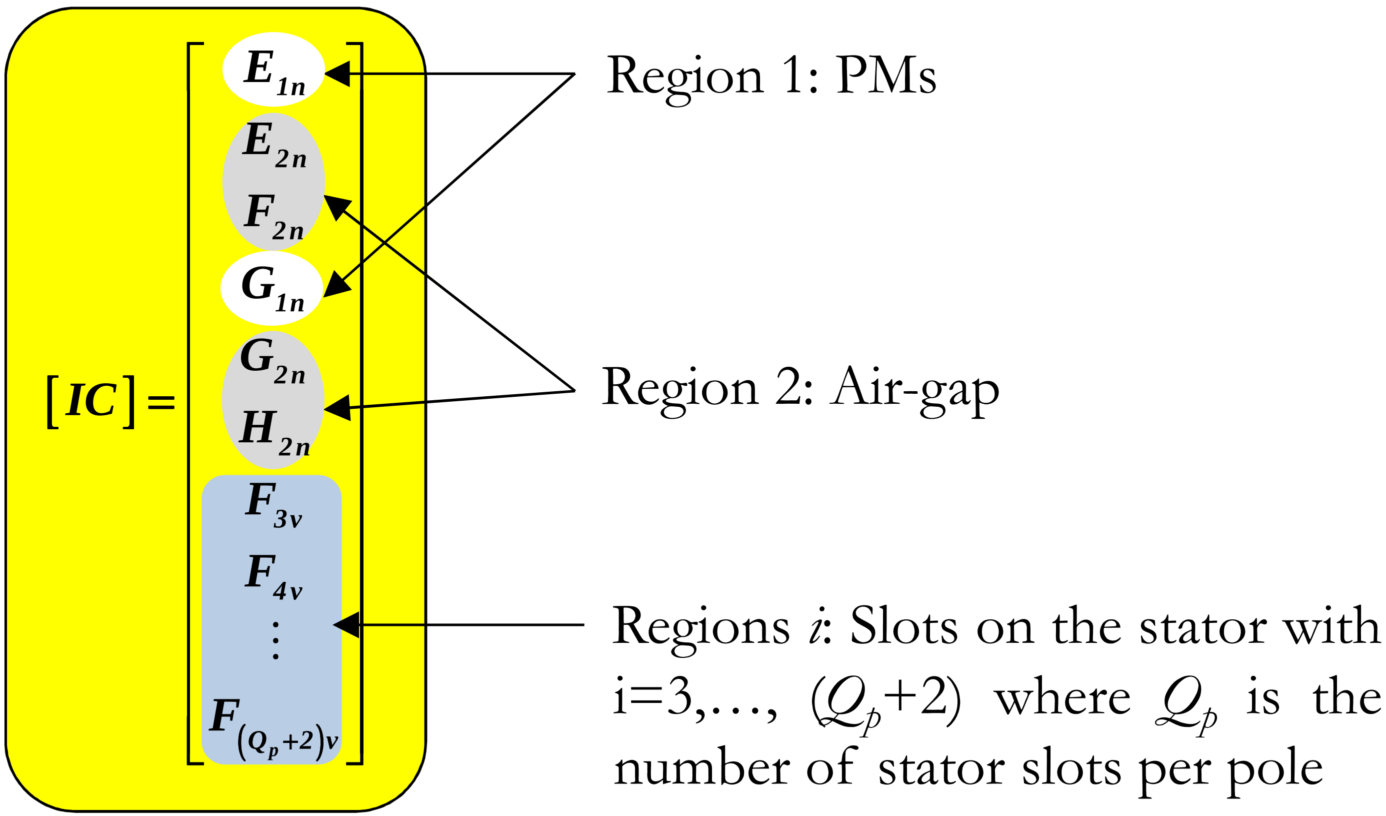
❑ Integration Constants [*IC*]:



2-D Exact Sub-Domain Model

- **Magnetic sources [*K*]:**

$$[K] = \begin{bmatrix} K_{1n} \\ K_{3n} \\ 0 \\ K_{2n} \\ K_{4n} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

with the matrices $K_{1n} \sim K_{4n}$ have $Nn \times 1$ coefficients which depend on Θ_{rs} .

2-D Exact Sub-Domain Model

❑ Boundary conditions matrix [*Q*]:

$$[Q]=\begin{bmatrix} Q_A & 0 & Q_B \\ 0 & Q_A & Q_C \\ Q_D & Q_E & Q_F \end{bmatrix}$$

$$Q_A=\begin{bmatrix} Q_{1nn} & Q_{0nn} & Q_{0nn} \\ Q_{2nn} & -Q_{0nn} & Q_{0nn} \\ 0 & Q_{3nn} & Q_{4nn} \end{bmatrix}$$

Q_{0nn} : the **unit matrix** has $Nn \times Nn$ coefficients.

$Q_{1nn} \sim Q_{4nn}$: the **diagonal matrices** have $Nn \times Nn$ coefficients.

2-D Exact Sub-Domain Model

❑ Boundary conditions matrix [*Q*]:

$$[Q] = \begin{bmatrix} Q_A & 0 & Q_B \\ 0 & Q_A & Q_C \\ Q_D & Q_E & Q_F \end{bmatrix}$$

$$Q_B = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ Q_{5nv3} & Q_{5nv4} & \cdots & Q_{5nv(Q_p+2)} \end{bmatrix} \qquad Q_C = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ Q_{6nv3} & Q_{6nv4} & \cdots & Q_{6nv(Q_p+2)} \end{bmatrix}$$

$$Q_D = \begin{bmatrix} 0 & Q_{7vn3} & Q_{8vn3} \\ 0 & Q_{7vn4} & Q_{8vn4} \\ \vdots & \vdots & \vdots \\ 0 & Q_{7vn(Q_p+2)} & Q_{8vn(Q_p+2)} \end{bmatrix} \qquad Q_E = \begin{bmatrix} 0 & Q_{9vn3} & Q_{10vn3} \\ 0 & Q_{9vn4} & Q_{10vn4} \\ \vdots & \vdots & \vdots \\ 0 & Q_{9vn(Q_p+2)} & Q_{10vn(Q_p+2)} \end{bmatrix}$$

$Q_{5nvi} \sim Q_{6nvi}$ and $Q_{7vni} \sim Q_{10vni}$: the **matrices** have $Nv \times Nn$ and $Nn \times Nv$ coefficients respectively.

2-D Exact Sub-Domain Model

❑ Boundary conditions matrix [*Q*]:

$$[Q] = \begin{bmatrix} Q_A & 0 & Q_B \\ 0 & Q_A & Q_C \\ Q_D & Q_E & Q_F \end{bmatrix}$$

$$Q_F = \begin{bmatrix} Q_{11vv3} & 0 & \cdots & 0 \\ 0 & Q_{11vv4} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_{11vv(Q_p+2)} \end{bmatrix}$$

Q_{11vvi} : the **diagonal matrix** has $Nv \times Nv$ coefficients.

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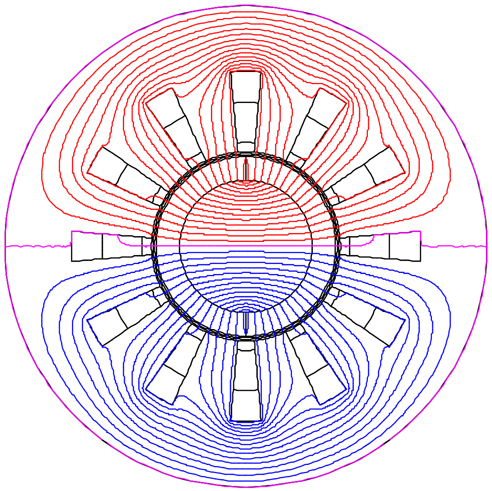
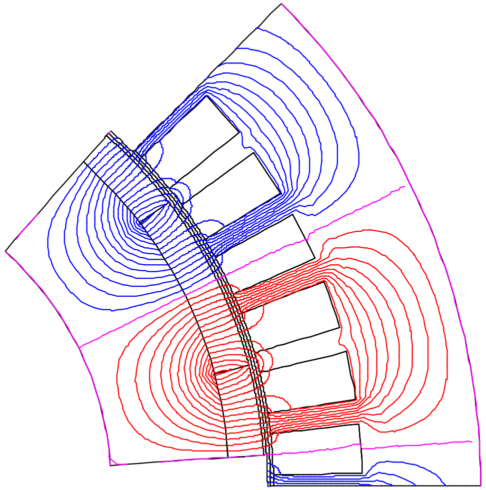


Motor 1: M1



Motor 2: M2

Parameters	M1	M2
Magnetization (R: Radial; P: Parallel)	R and P	
Number of pole pairs, **p** [–]	1	8
Total number of slots, Q_s [–]	12	48
Number of stator slots per pole, Q_p [–]	6	3
Magnet pole-arc to pole-pitch ratio, $\alpha_p=\Theta_m/\Theta_p$ [%]	100	
Stator slot opening to tooth-pitch ratio, $\zeta_o=\Theta_o/\Theta_t$ [%]	33.33	66.67
Radius of the stator yoke surface, R_{sy} [mm]	37	190
Radius of the stator surface, R_s [mm]	20	160
Radius of the PMs surface, R_m [mm]	19	157
Radius of the rotor yoke surface, R_r [mm]	14	145
Axial length, L [mm]	45	180
Remanent flux density of the PMs, B_{rm} [T]	1.13	
Relative magnetic permeability of the PMs, μ_{rm} [–]	1.029	

Analytical Model

$Nv = 25$

$Nn = 50$

Motor 1: 450 éléments

Motor 2: 375 éléments

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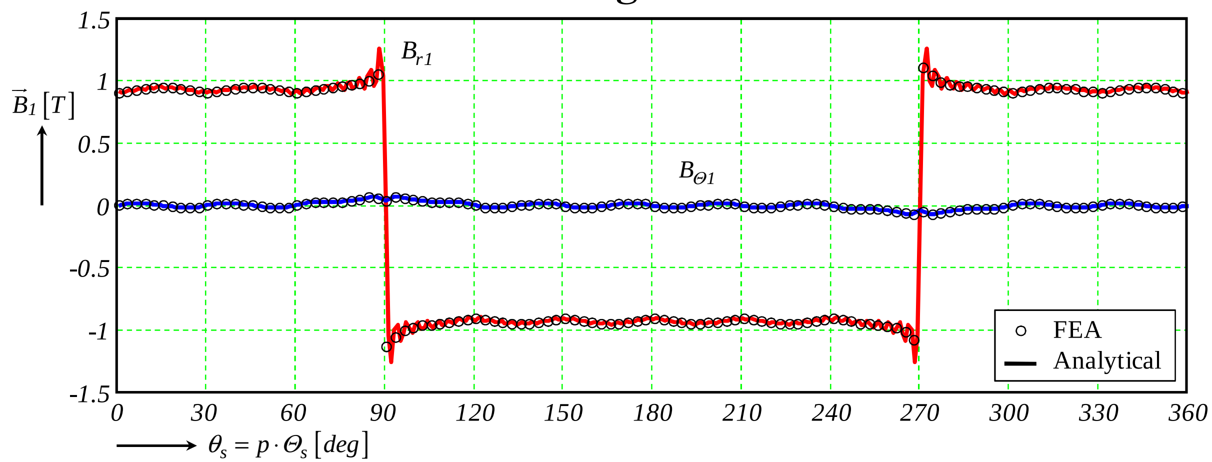
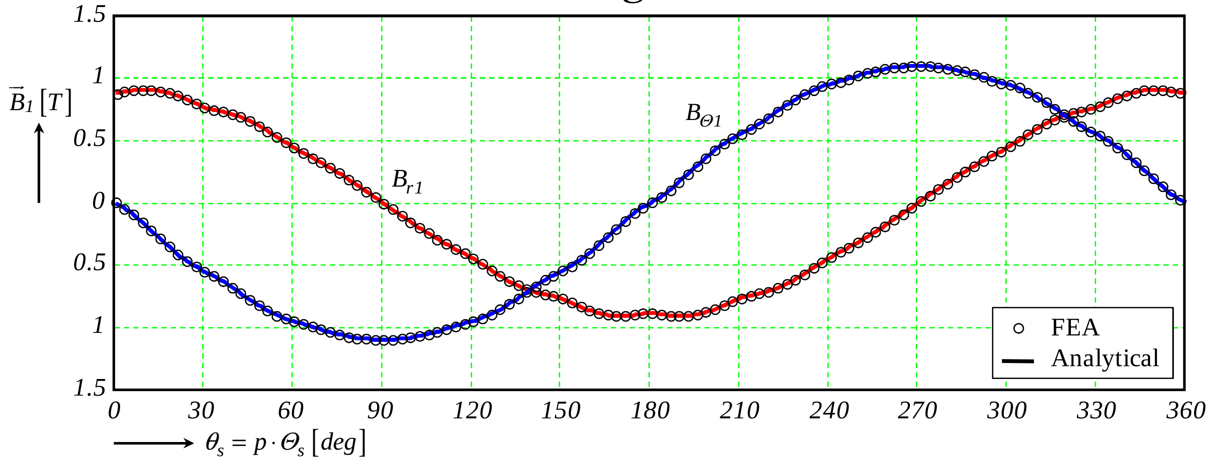
❑ Motor 1: B_{r1} and $B_{\Theta 1}$ in the middle of the PMs

Radial magnetized PMs



Parallel magnetized PMs



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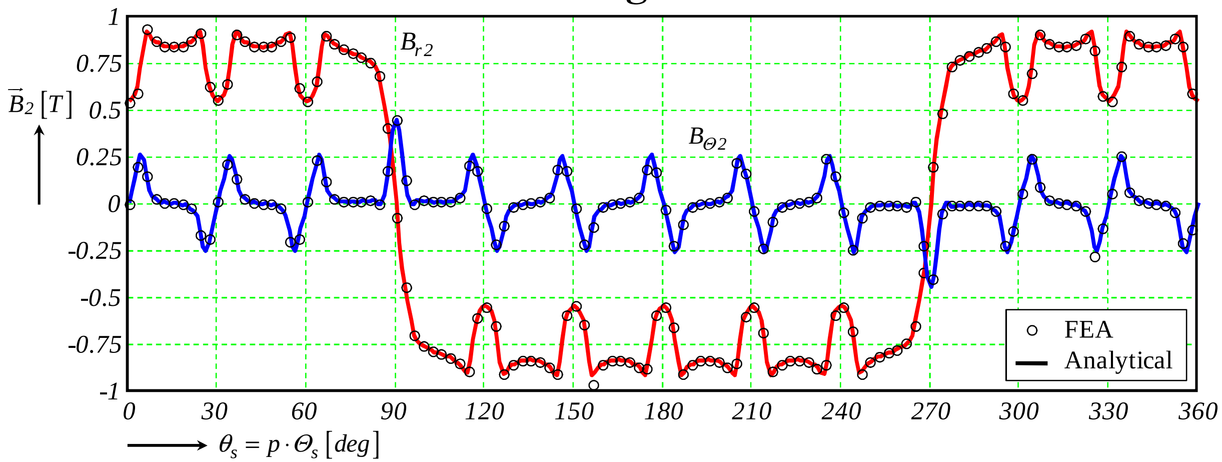
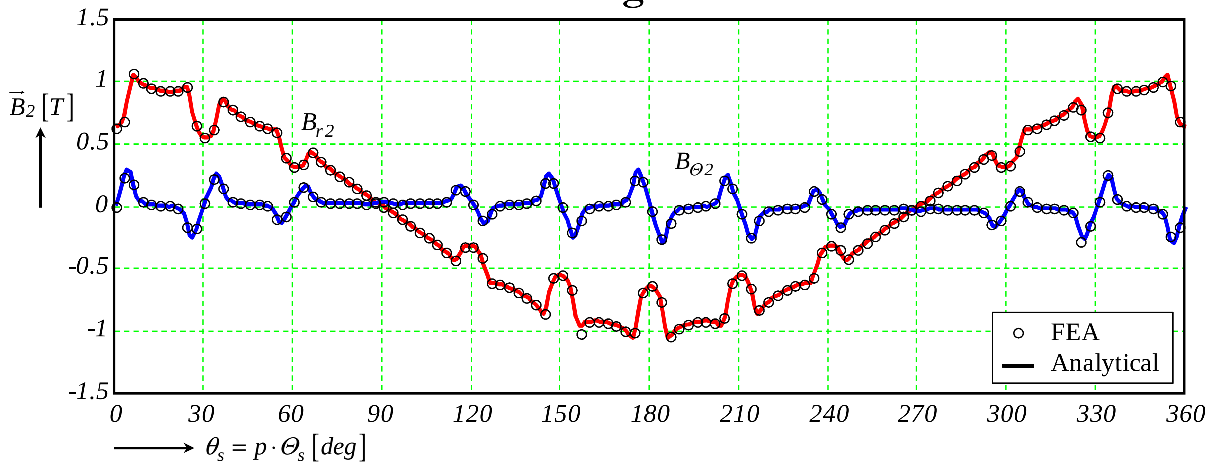
Motor 1: B_{r2} and $B_{\Theta 2}$ in the middle of the air-gap

Radial magnetized PMs



Parallel magnetized PMs



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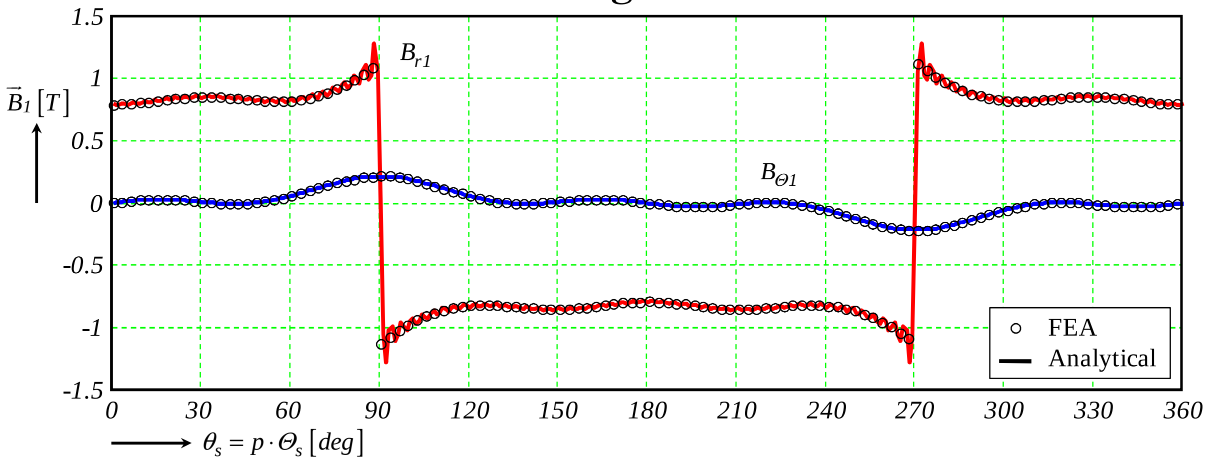
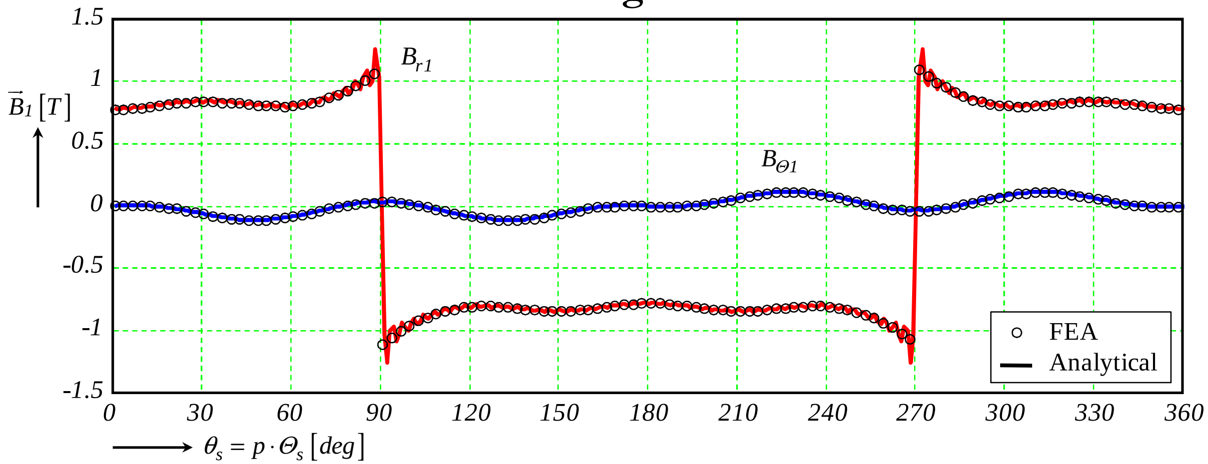
Motor 2: B_{r1} and $B_{\Theta 1}$ in the middle of the PMs

Radial magnetized PMs



Parallel magnetized PMs



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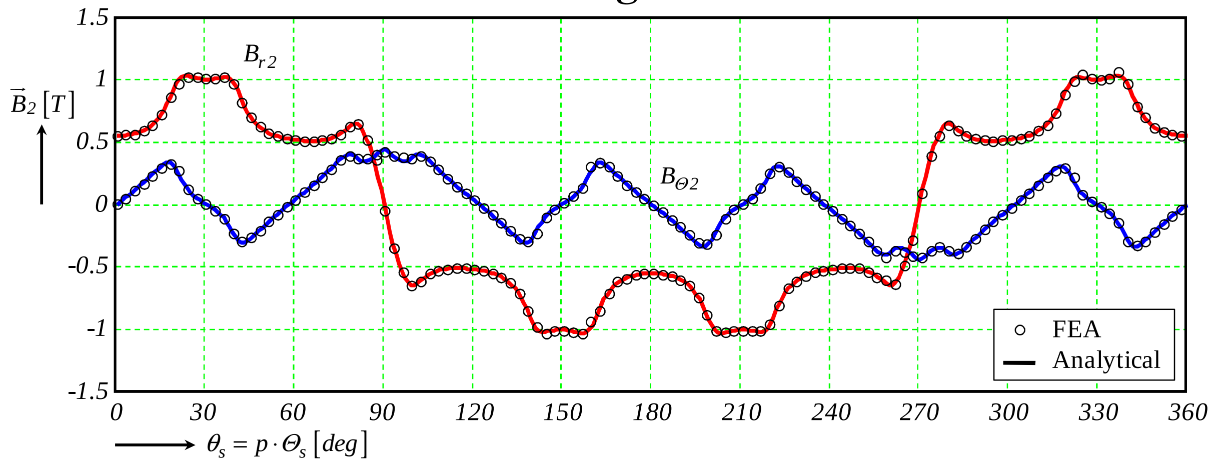
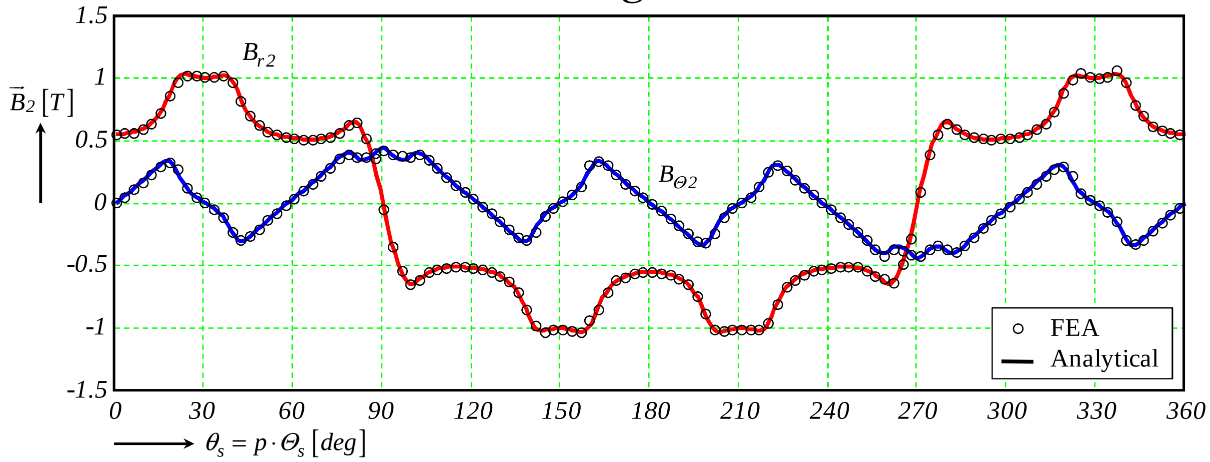
Radial magnetized PMs



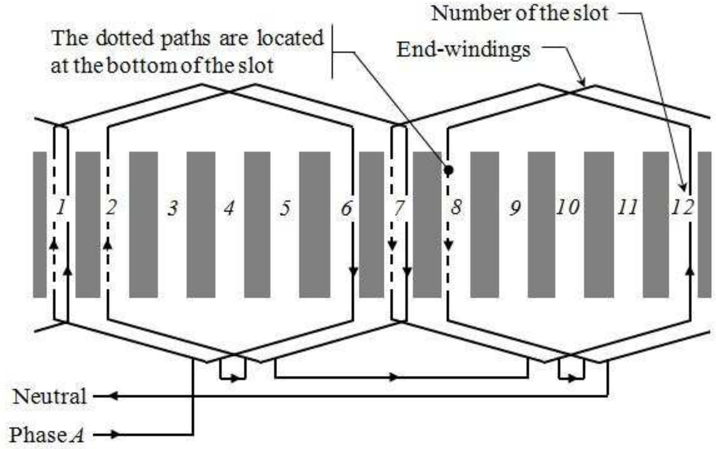
Parallel magnetized PMs



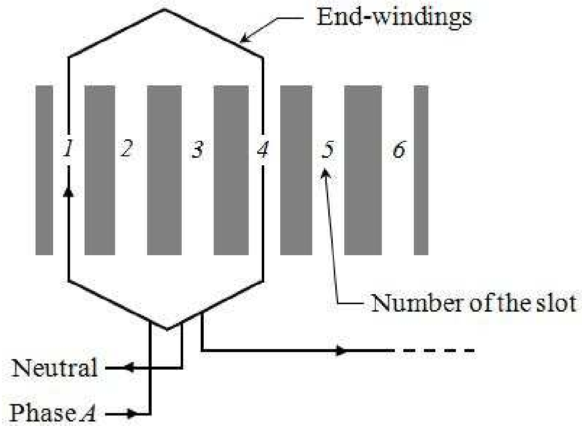
Back Electromotive Force (EMF)

- **Spatial Distribution of the Overlapping *3*-phase stator windings:**

Motor 1: N_l=2, N=48, α_{wp}=5/6



Motor 2: N_l=1, N=48, α_{wp}=1



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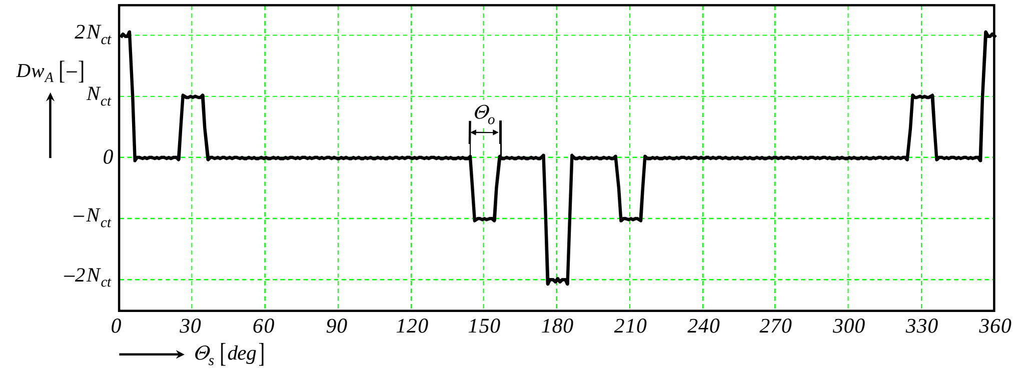
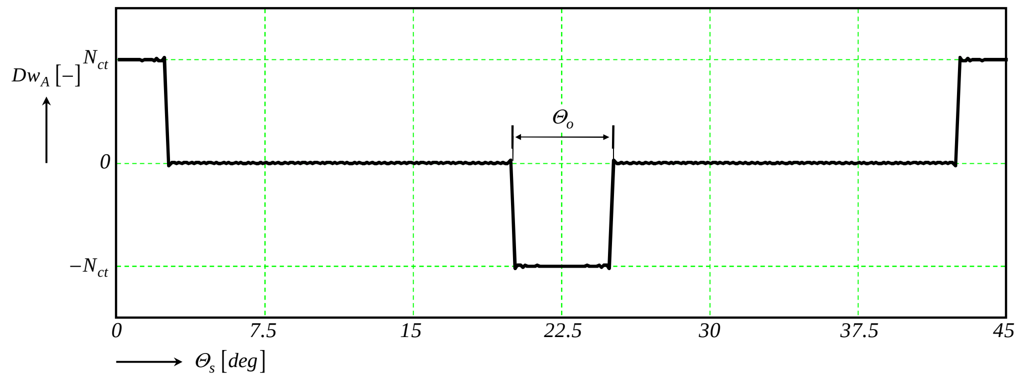


Motor 2:



$$Dw_g = \sum_{n\ odd}^{\infty} Dw_n \cdot \cos\left[np \cdot \left(\Theta_s - g \cdot \frac{2\Theta_p}{m}\right)\right] \quad \text{with} \quad Dw_n = 2\alpha_o \cdot \Theta_p \cdot \frac{N \cdot K_w(np)}{\pi}$$

Back Electromotive Force (EMF)

❑ Faraday's Law:

$$E_g = -\frac{d\psi_g}{dt} = -\Omega_0 \cdot \frac{d\psi_g}{d\Theta_{rs}} \quad \text{with} \quad \psi_g = 2p \cdot L \cdot \int_{-\Theta_p}^{\Theta_p} \frac{Dw_g}{\left(\alpha_o \cdot \Theta_p\right)^2} \cdot A_{z2}\Big|_{r=R_s} \cdot d\Theta_s$$

❑ Analytical Solution of the back EMF:

$$\boldsymbol{E_g = N_0 \cdot S_{cyl} \cdot B_{rm} \cdot f_E}$$

with $S_{cyl} = 2\pi \cdot R_s \cdot L$

$$f_E = -\sum_{n\ odd}^{\infty} Dw_n \cdot f_{Ebn} \cdot \left[Es_n \cdot \sin\left(np \cdot g \cdot \frac{2\Theta_p}{m} \right) + Ec_n \cdot \cos\left(np \cdot g \cdot \frac{2\Theta_p}{m} \right) \right]$$

$$Es_n = E'_{2n} \cdot f_{Ean} + F'_{2n} \quad \& \quad Ec_n = G'_{2n} \cdot f_{Ean} + H'_{2n}$$

$$f_{Ean} = 1 \Big/ \left(\frac{R_m}{R_s} \right)^{2 \cdot np} \quad \& \quad f_{Ebn} = \frac{\pi}{30} \cdot \frac{1}{2\alpha_o \cdot \Theta_p} \cdot \left(\frac{R_m}{R_s} \right)^{np+1}$$

Comp.: Analytical Solution vs FEA 2-D

- *Context and Objective of the Work*
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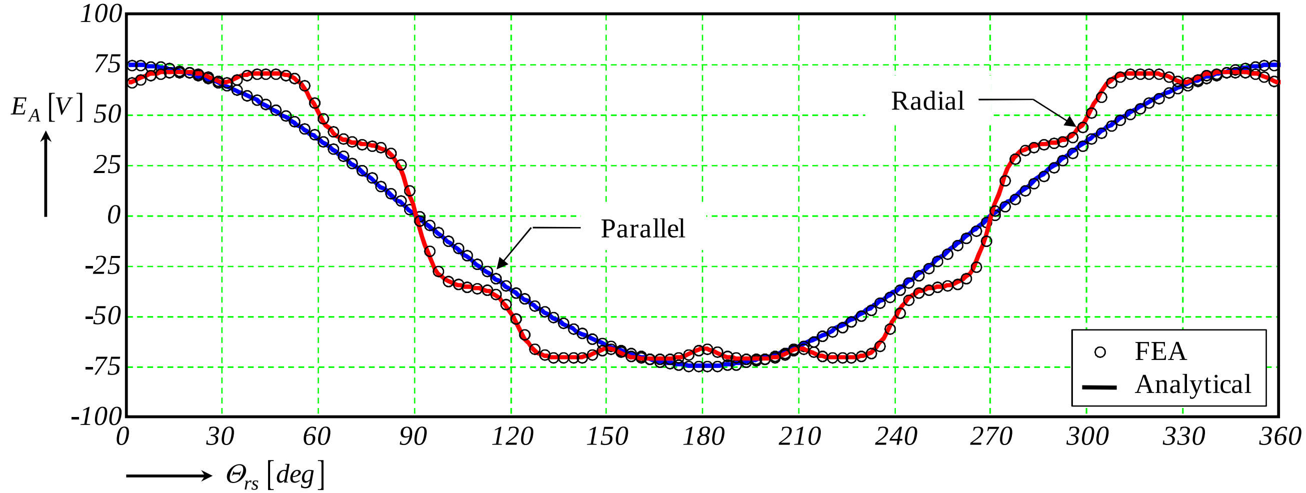
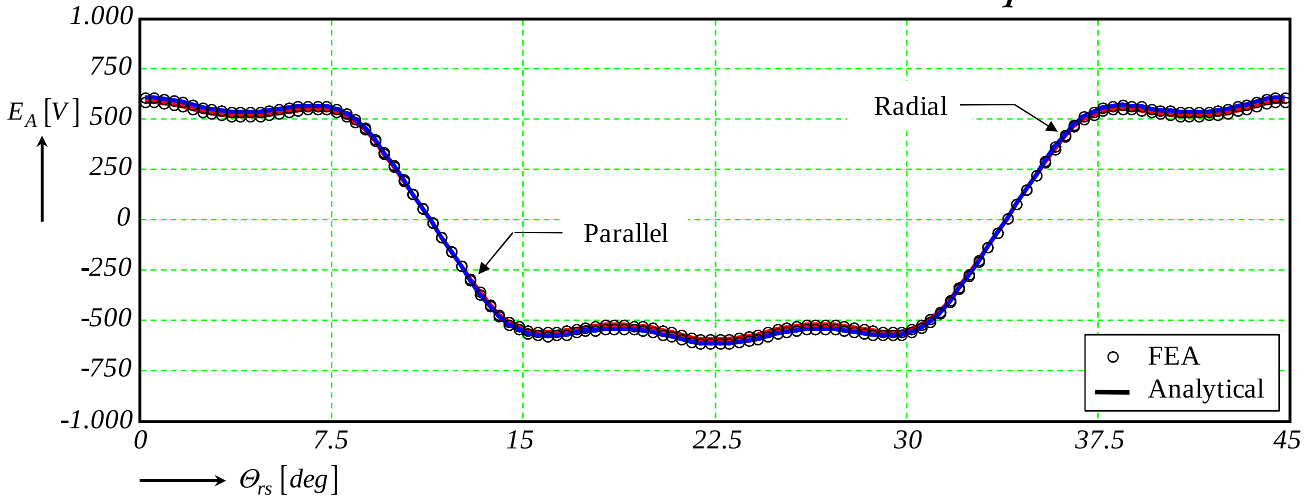
- ***Full Quantities No-Load Calculation***
- *Conclusion*

- **Motor 1: p=1, Q_s=12, N_l=2, N=48, α_{wp}=5/6**



- **Motor 2: p=8, Q_s=48, N_l=1, N=48, α_{wp}=1**



Cogging Torque Calculation

- **Maxwell Stress Tensor:**

$$T_c = -\frac{p \cdot R_s}{\mu_0} \cdot \int_0^L \int_{-\Theta_p}^{\Theta_p} \frac{\partial A_{z2}}{\partial r} \cdot \frac{\partial A_{z2}}{\partial \Theta_s}\bigg|_{r=R_s} \cdot d\Theta_s \cdot dz$$

- **Analytical Solution of the Cogging Torque:**

$$\boldsymbol{T_c = V_{cyl} \cdot \frac{B_{rm}^{\,2}}{\mu_0} \cdot f_T}$$

with $V_{cyl} = \pi \cdot R_s^{\,2} \cdot L$

$$f_T = \sum_{n\ odd}^{\infty} f_{Tn} \cdot \left(E_{2n} \cdot H_{2n} - F_{2n} \cdot G_{2n}\right) \qquad \text{where} \qquad f_{Tn} = 2 \cdot \left(np \cdot \frac{R_m}{R_s}\right)^2$$

- **representing physically the air-gap reluctances;**
- **is equal to 0 for the slotless motors: the cogging torque does not existed.**

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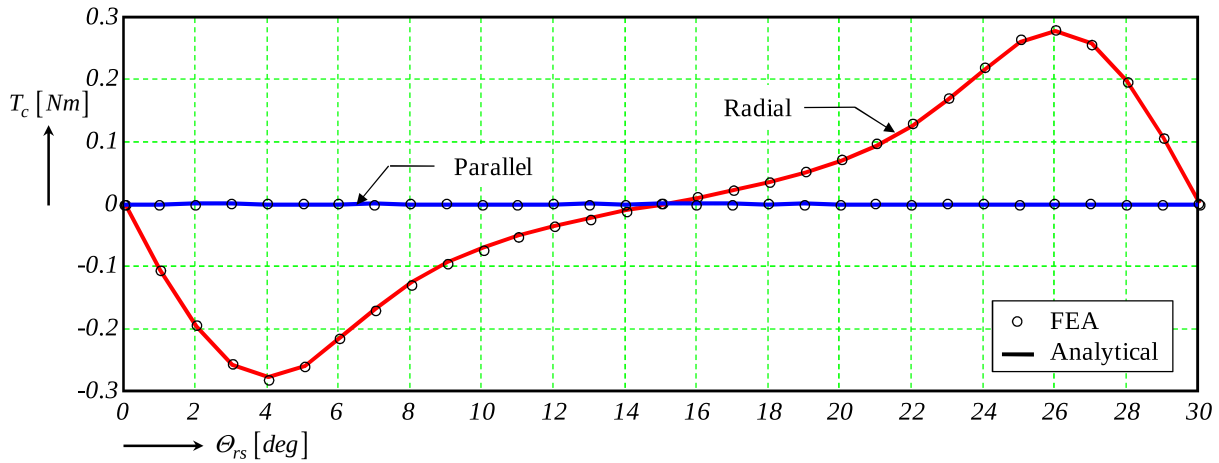
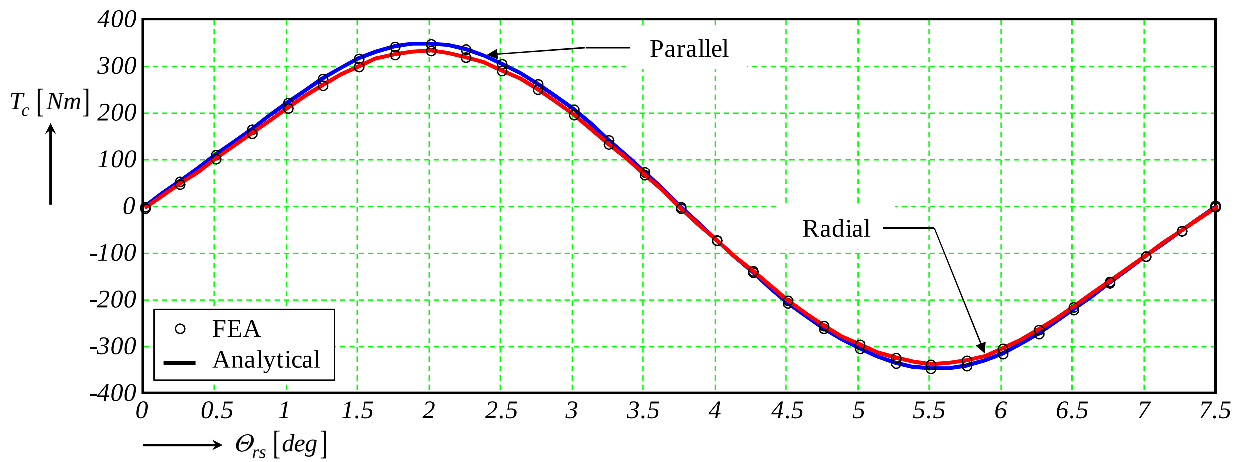


Motor 2: p=8, Q_s=48, ζ_o=66.67%



2-D PMs Eddy-Current Losses

- **Eddy-currents density and losses:**

$$J_{z1} = -\sigma_m \cdot \Omega_0 \cdot \frac{\partial A_{z1}}{\partial \Theta_{rs}} + C \quad \text{and} \quad p_m^{slot} = p \cdot L \cdot \int_{R_r}^{R_m} \int_{-\Theta_p}^{\Theta_p} \frac{J_{z1}^{\ 2}}{\sigma_m} \cdot r \cdot dr \cdot d\Theta_r$$

- **Analytical Solution of No-Load Losses:**

$$\boldsymbol{P_m^{slot} = k_{nsfd} \cdot k_e \cdot N_0^{\ 2} \cdot B_{rm}^{\ 2} \cdot M_m}$$

with M_m : Mass of the PMs

$k_e = \dfrac{\pi^2 \cdot \sigma_m \cdot L^2}{6 \cdot \rho_{vm}}$: Eddy-current losses coefficient in the PMs

$k_{nsfd} = f\left(E'_{2n}, G'_{2n}, E_{2n}, G_{2n}\right)$

↓

- **representing physically the non-sinusoidal magnetic flux density produced by the PMs;**
- **is equal to 0 for the slotless motors: the 2-D eddy-current losses does not existed.**

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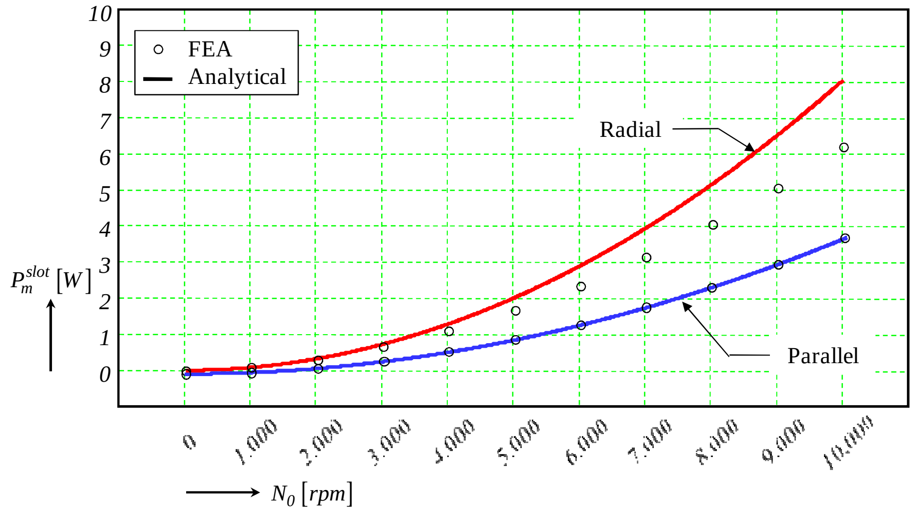
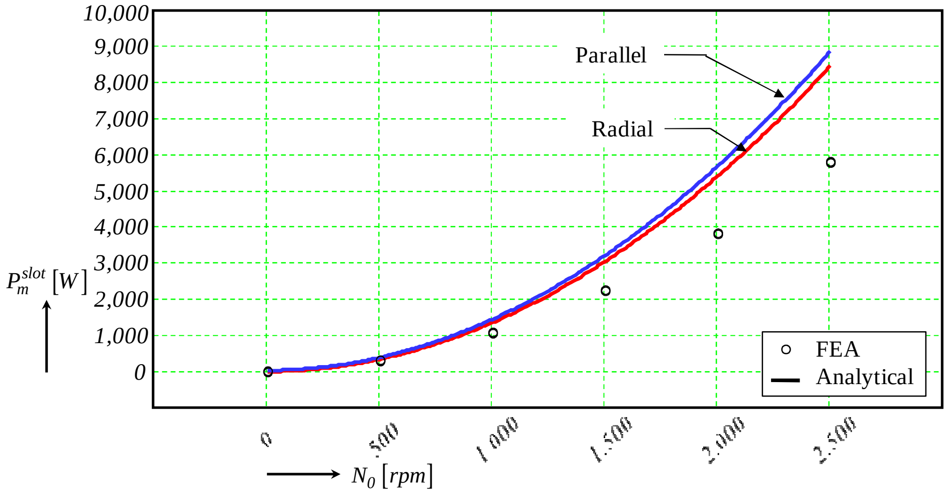


Motor 2: p=8, Q_s=48, ζ_o=66.67%



Conclusion

- **Very good results (the local an integral quantities) excluding the PMs eddy-current losses** ⇨ These differences are due to eddy-current effect on the magnetic field which is not taken into account in 2-D exact sub-domain model.

- **Less computing time than the FEA 2-D.**

- **More rigorous than the methods based on the Schwarz-Christoffel transformation or the permeance network.**

- **A useful tool for design and optimization of multi-pole SMPMM.**

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