Slotting Effect in Permanent-Magnet Motors via a 2-D Exact Sub-Domain Model

F. Dubas and C. Espanet
Context and Objective of the Work

Review of the Existing Analytical Models

2-D Analytical Field Model (i.e., Sub-Domains)

Full Quantities No-Load Calculation

Conclusion
Introduction: Context and Objective

- Accurate knowledge of the magnetic field distribution in the air-gap.
- Multi-pole surface mounted permanent-magnet (PM) motors (SMPMM).
- Modeling of slotting effect.
- Full quantities no-load evaluation.
- New 2-D analytical Solution.
- Comparison analytical model vs Finite Element Analysis (FEA 2-D)
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Review of the Existing Analytical Models

**Electromagnetic Modeling**

- **Numerical Method**
- **Analytical Method**
- **Permeance Network**

Maxwell's equations

- **1-D Analytical Model.**
- **2-D Slotless Analytical Model [1].**
- **Slotting Effect:**
  1. Conformal Transformation (Relative or Complex Permeance Model);
  2. Exact Sub-Domain [2].

Slotting Effect in Permanent-Magnet Motors via a 2-D Exact Sub-Domain Model

Problem Description and Assumptions

- End-effects are neglected.
- Saturation is neglected.
- Non-conductive PMs material (i.e., non-resolution of Diffusion's equations).
- Radial/Parallel magnetization.
- Linear demagnetization characteristics of PMs.
- Radial slot faces on the stator.
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2-D Exact Sub-Domain Model

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Region 1: PMs
Region 2: Air-gap
Regions $i$: Slots on the stator with $i = 3, \ldots, (Q_p + 2)$

$Q_p$: Number of stator slots per pole

- Poisson's equations: $\Delta A_{z1} = B_{rm} \cdot K_m (r, \Theta_s)$ in the PMs.

- Laplace's equations: $\Delta A_{z2} = 0$ in the air-gap.
  $\Delta A_{zi} = 0$ in the slots on the stator.
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In the PMs: \( A_{z1} = B_{rm} \cdot R_m \cdot f_{z1} (E_{1n}, G_{1n}, r, \Theta_s) \)

In the air-gap: \( A_{z2} = B_{rm} \cdot R_m \cdot f_{z2} (E_{2n} \sim H_{2n}, r, \Theta_s) \)

In the slots on the stator: \( A_{zi} = B_{rm} \cdot R_s \cdot f_{zi} (F_{iv}, r, \Theta_s) \)
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2-D Exact Sub-Domain Model

- The linear Cramer's system:

\[
[IC] = [Q]^{-1} \cdot [K]
\]

\begin{itemize}
  \item Integration Constants
  \item Boundary conditions matrix
  \item Magnetic sources (i.e., PM and Current)
\end{itemize}

- Size of the Cramer's system, i.e., \([Q]\):

\[6 \cdot Nn + Q_p \cdot Nv\] equations and unknowns.

with \(Nn\) and \(Nv\) are the number of terms in the Fourier's series for the computation \(A_{z1}\), \(A_{z2}\) and \(A_{zi}\)

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Integration Constants \([IC]\):

\[
[IC] = \begin{bmatrix}
E_{1n} \\
E_{2n} \\
F_{2n} \\
G_{1n} \\
G_{2n} \\
H_{2n} \\
F_{3v} \\
F_{4v} \\
\vdots \\
F_{(Q_p+2)v}
\end{bmatrix}
\]

Region 1: PMs

Region 2: Air-gap

Regions \(i\): Slots on the stator with \(i=3, \ldots, (Q_p+2)\) where \(Q_p\) is the number of stator slots per pole
Magnetic sources $[K]$:

$$[K] = \begin{bmatrix}
K_{1n} \\
K_{3n} \\
0 \\
K_{2n} \\
K_{4n} \\
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}$$

with the matrices $K_{1n} \sim K_{4n}$ have $N \times 1$ coefficients which depend on $\Theta_{rs}$. 
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2-D Exact Sub-Domain Model

- **Boundary conditions matrix** $[Q]$: 

$$
[Q] = \begin{bmatrix}
Q_A & 0 & Q_B \\
0 & Q_A & Q_C \\
Q_D & Q_E & Q_F \\
\end{bmatrix}
$$

- $Q_A = \begin{bmatrix}
Q_{1nn} & Q_{0nn} & Q_{0nn} \\
Q_{2nn} & -Q_{0nn} & Q_{0nn} \\
0 & Q_{3nn} & Q_{4nn} \\
\end{bmatrix}$

$Q_{0nn}$: the **unit matrix** has $Nn \times Nn$ coefficients.

$Q_{1nn} \sim Q_{4nn}$: the **diagonal matrices** have $Nn \times Nn$ coefficients.
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Boundary conditions matrix $[Q]$:

$$
[Q] = \begin{bmatrix}
Q_A & 0 & Q_B \\
0 & Q_A & Q_C \\
Q_D & Q_E & Q_F
\end{bmatrix}
$$

- $Q_B = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
Q_{5nv3} & Q_{5nv4} & \cdots & Q_{5nv(Q_p+2)}
\end{bmatrix}$
- $Q_C = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
Q_{6nv3} & Q_{6nv4} & \cdots & Q_{6nv(Q_p+2)}
\end{bmatrix}$
- $Q_D = \begin{bmatrix}
0 & Q_{7vn3} & Q_{8vn3} \\
0 & Q_{7vn4} & Q_{8vn4} \\
\vdots & \vdots & \vdots \\
0 & Q_{7vn(Q_p+2)} & Q_{8vn(Q_p+2)}
\end{bmatrix}$
- $Q_E = \begin{bmatrix}
0 & Q_{9vn3} & Q_{10vn3} \\
0 & Q_{9vn4} & Q_{10vn4} \\
\vdots & \vdots & \vdots \\
0 & Q_{9vn(Q_p+2)} & Q_{10vn(Q_p+2)}
\end{bmatrix}$

$Q_{5nvi} \sim Q_{6nvi}$ and $Q_{7nvi} \sim Q_{10nvi}$: the matrices have $Nv \times Nn$ and $Nn \times Nv$ coefficients respectively.
Boundary conditions matrix $[Q]$:

\[
Q_F = \begin{bmatrix}
Q_{11vv3} & 0 & \cdots & 0 \\
0 & Q_{11vv4} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & Q_{11vv(N_v+2)} \\
\end{bmatrix}
\]

$Q_{11vv}$: the **diagonal matrix** has $N_v \times N_v$ coefficients.
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Motor 1: M1

Motor 2: M2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>M1</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetization (R: Radial; P: Parallel)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of pole pairs, p [-]</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Total number of slots, Q_s [-]</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>Number of stator slots per pole, Q_p [-]</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Magnet pole-arc to pole-pitch ratio, ( a_p = \Theta_m / \Theta_p ) [%]</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Stator slot opening to tooth-pitch ratio, ( \zeta_o = \Theta_o / \Theta_t ) [%]</td>
<td>33.33</td>
<td>66.67</td>
</tr>
<tr>
<td>Radius of the stator yoke surface, ( R_{sy} ) [mm]</td>
<td>37</td>
<td>190</td>
</tr>
<tr>
<td>Radius of the stator surface, ( R_s ) [mm]</td>
<td>20</td>
<td>160</td>
</tr>
<tr>
<td>Radius of the PMs surface, ( R_m ) [mm]</td>
<td>19</td>
<td>157</td>
</tr>
<tr>
<td>Radius of the rotor yoke surface, ( R_r ) [mm]</td>
<td>14</td>
<td>145</td>
</tr>
<tr>
<td>Axial length, ( L ) [mm]</td>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>Remanent flux density of the PMs, ( B_{rm} ) [T]</td>
<td></td>
<td>1.13</td>
</tr>
<tr>
<td>Relative magnetic permeability of the PMs, ( \mu_{rm} ) [-]</td>
<td>1.029</td>
<td></td>
</tr>
</tbody>
</table>

Analytical Model

\( N_v = 25 \)
\( N_n = 50 \)

Motor 1: 450 éléments
Motor 2: 375 éléments
Motor 1: $B_{rl}$ and $B_{\Theta l}$ in the middle of the PMs

Radial magnetized PMs

Parallel magnetized PMs

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Motor 1: $B_{r2}$ and $B_{\Theta2}$ in the middle of the air-gap

Radial magnetized PMs

Parallel magnetized PMs
Motor 2: $B_{rl}$ and $B_{\Theta l}$ in the middle of the PMs

Radial magnetized PMs

Parallel magnetized PMs
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Comp.: Analytical Model vs FEA 2-D

Motor 2: $B_{r2}$ and $B_{\Theta2}$ in the middle of the air-gap

Radial magnetized PMs

Parallel magnetized PMs

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Back Electromotive Force (EMF)

- Spatial Distribution of the Overlapping 3-phase stator windings:

Motor 1: $N_f=2$, $N=48$, $\alpha_{wp}=5/6$

Motor 2: $N_f=1$, $N=48$, $\alpha_{wp}=1$
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Back Electromotive Force (EMF)

- Spatial Distribution of the Overlapping 3-phase stator windings:

Motor 1:

Motor 2:

\[ Dw_g = \sum_{n \text{ odd}} Dw_n \cdot \cos \left( np \left( \Theta_s - g \frac{2\Theta_p}{m} \right) \right) \]

with \[ Dw_n = 2\alpha_o \cdot \Theta_p \cdot \frac{N \cdot K_w (np)}{\pi} \]
Faraday's Law:

\[ E_g = -\frac{d\psi_g}{dt} = -\Omega_0 \cdot \frac{d\psi_g}{d\Theta_{rs}} \quad \text{with} \quad \psi_g = 2p \cdot L \cdot \int_{-\Theta_p}^{\Theta_p} \frac{Dw_g}{(\alpha_o \cdot \Theta_p)^2} \cdot A_{\zeta, 2} \left|_{r = r_s} \right. \cdot d\Theta_s \]

Analytical Solution of the back EMF:

\[ E_g = N_0 \cdot S_{cyl} \cdot B_{rm} \cdot f_E \]

with \( S_{cyl} = 2\pi \cdot R_s \cdot L \)

\[ f_E = -\sum_{n \text{ odd}}^{\infty} Dw_n \cdot f_{Ebn} \cdot \left[ E_{s_n} \cdot \sin \left( np \cdot g \cdot \frac{2\Theta_p}{m} \right) + E_{c_n} \cdot \cos \left( np \cdot g \cdot \frac{2\Theta_p}{m} \right) \right] \]

\[ E_{s_n} = E_{2n}^s \cdot f_{Ean} + F_{2n}^s \quad \& \quad E_{c_n} = G_{2n}^c \cdot f_{Ean} + H_{2n}^c \]

\[ f_{Ean} = \frac{1}{\left( \frac{R_m}{R_s} \right)^{2np}} \quad \& \quad f_{Ebn} = \frac{\pi \cdot 1}{30 \cdot 2\alpha_o \cdot \Theta_p \cdot \left( \frac{R_m}{R_s} \right)^{np+1}} \]
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Comp.: Analytical Solution vs FEA 2-D

Motor 1: \( p=1, Q_s=12, N_I=2, N=48, \alpha_{wp}=5/6 \)

Motor 2: \( p=8, Q_s=48, N_I=1, N=48, \alpha_{wp}=1 \)

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Cogging Torque Calculation

- **Maxwell Stress Tensor:**

  \[ T_c = -\frac{p \cdot R_s}{\mu_0} \int_{0}^{L} \int_{-\theta_p}^{\theta_p} \frac{\partial A_{z2}}{\partial r} \cdot \frac{\partial A_{z2}}{\partial \Theta_s} \bigg|_{r=R_s} \cdot d\Theta_s \cdot dz \]

- **Analytical Solution of the Cogging Torque:**

  \[ T_c = V_{cyl} \cdot \frac{B_{rm}^2}{\mu_0} \cdot f_T \]

  with \( V_{cyl} = \pi \cdot R_s^2 \cdot L \)

  \[ f_T = \sum_{n, odd} f_{Tn} \cdot (E_{2n} \cdot H_{2n} - F_{2n} \cdot G_{2n}) \]  
  where \( f_{Tn} = 2 \cdot \left( np \cdot \frac{R_m}{R_s} \right)^2 \)

  - representing physically the air-gap reluctances;
  - is equal to 0 for the slotless motors: the cogging torque does not existed.
Motor 1: $p=1$, $Q_s=12$, $\zeta_o=33.33\%$

Motor 2: $p=8$, $Q_s=48$, $\zeta_o=66.67\%$
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2-D PMs Eddy-Current Losses

- Eddy-currents density and losses:

\[
J_{z1} = -\sigma_m \cdot \Omega_0 \cdot \frac{\partial A_{z1}}{\partial \Theta_{rs}} + C \quad \text{and} \quad p_m^{\text{slot}} = p \cdot L \cdot \int_{r_p}^{r_m} \int_{-\Theta_p}^{\Theta_p} \frac{J_{z1}^2}{\sigma_m} \cdot r \cdot dr \cdot d\Theta_r
\]

- Analytical Solution of No-Load Losses:

\[
P_m^{\text{slot}} = k_{nsfd} \cdot k_e \cdot N_0^2 \cdot B_{rm}^2 \cdot M_m
\]

with \(M_m\) : Mass of the PMs

\[
k_e = \frac{\pi^2 \cdot \sigma_m \cdot L^2}{6 \cdot \rho_{vm}} \quad \text{: Eddy-current losses coefficient in the PMs}
\]

\[
k_{nsfd} = f(E'_{2n}, G'_{2n}, E_{2n}, G_{2n})
\]

- representing physically the non-sinusoidal magnetic flux density produced by the PMs;
- is equal to 0 for the slotless motors: the 2-D eddy-current losses does not existed.

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Motor 2: \( p=8, Q_s=48, \zeta_o=66.67\% \)
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Conclusion

- Very good results (the local an integral quantities) excluding the PMs eddy-current losses ⇒ These differences are due to eddy-current effect on the magnetic field which is not taken into account in 2-D exact sub-domain model.

- Less computing time than the FEA 2-D.

- More rigorous than the methods based on the Schwarz-Christoffel transformation or the permeance network.

- A useful tool for design and optimization of multi-pole SMPMM.
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