Nonlinear Dynamics of a Hybrid Piezo-electromagnetic Vibrating Energy Harvester

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Abstract

The nonlinear dynamics of a novel hybrid piezo-electromagnetic vibrating energy harvester is modeled and investigated. The proposed concept takes advantage of the mechanical elasticity of a sandwich PZT/Steel/PZT beam to perform a vertical guidance for a vibrating magnet while combining electromagnetic and piezoelectric transduction techniques at large displacement. We follow the extended Hamilton principle in order to derive the mutiphysics continuum problem and discretize it into a finite system of nonlinear ordinary differential equations in time domain using the Galerkin method. The resulting reduced order model is solved numerically using the harmonic balance method coupled with the asymptotic numerical continuation technique. Several numerical simulations have been performed showing that the performances of a classical vibrating energy harvester can be significantly enhanced up to $100\,\%$ in term of power density and up to $10\,\%$ in term of frequency bandwidth.

Keywords: *Energy harvesting, nonlinear dynamic, hybrid harvester, magnetic levitation, piezoelectric generator.*

1 Introduction

Energy harvesting from ambient energy aims at realizing electromechanical generators to supply autonomous microsystems from energy of their local environment. The research motivation in energy harvesting is related to the willingness to measure, monitor, and process data from a hostile environment, and be able to communicate in a completely autonomous way. Also, it is due to the reduced power requirement of these small harvester components. On this concept, vibration energy harvesting provides an efficient solution to implement self-sustained low-power microelectromechanical systems or MEMS. In order to make this mechanical energy of ambient vibration usable and turn it into useful electrical energy, there are several types of electromechanical transduction, where the most common transduction modes are electromagnetic [1] and piezoelectric [2].

Conventional linear vibration energy harvesters are usually designed to be resonantly tuned to the ambient dominant frequency. When the system's resonance and excitation frequency do not coincide, the linear harvesting devices are known to under perform. Also, for these kind of devices, the energy from multi-frequency or broad-band excitation may be not suitably captured. So, to increase the widening of the bandwidth and to have more harvested power, researches are oriented towards the study of nonlinear systems. Ferrari et al [3] proposed a nonlinear piezoelectric harvester that exploit stochastic resonance with white-noise excitation. For instance, Mann et al [4] took into consideration in their device, magnetic nonlinearity writing the magnetic force as a combination of linear and nonlinear stiffness. They demonstrated that the insertion of the system nonlinearity can result in relatively large oscillations over a wide frequency-band, *i.e.* the potential improvement of the ability to exploit ambient energy. Recently, new models of energy harvesting, called hybrid harvesters have been developed. These new devices use both transduction techniques, piezoelectric and electromagnetic explained previously. Vinod et al. [5] have developed a hybrid harvester and concluded that the total power is the sum of power produced the magnetic and the piezoelectric ways. Karami et al. [6] analyzed the influence of mechanical and electrical damping on the harvester power. They investigated also the effects of multiphysics coupling of a nonlinear hybrid harvester.

In this paper, the nonlinear dynamics of a novel hybrid piezo-electromagnetic vibrating energy harvester is modeled and

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investigated. The proposed concept takes advantage of the mechanical elasticity of a sandwich PZT/Steel/PZT beam to perform a vertical guidance for a vibrating magnet while combining electromagnetic and piezoelectric transduction techniques at large displacements.
The extended Hamilton principle is applied to the considered system in order to derive the equation of motion describing the vertical vibrations of the moving magnet while taking into account the geometric and magnetic nonlinearities. The Galerkin discretization procedure was used in order to transform the mutiphysics continuum problem into a finite system of nonlinear ordinary differential equations in time. The reduced order model is solved numerically by the harmonic balance method coupled with the asymptotic numerical continuation technique. Based on the proposed concept, several numerical simulations have been performed showing that the performances of a particular vibrating energy harvester [4] can be significantly enhanced in terms of power density and bandwidth.

2 Design and system modeling

The concept of the proposed design (Fig 1) is inspired from energy harvesters based on magnetic levitation as described in Refs [1, 4]. The main drawback of such electromagnetic harvesters is the large mechanical damping caused by the direct contact of the moving magnet with the inner surface of the tube which reduces the amplitude oscillation, *i.e* diminishes the harvested power. This requires the maintenance of the harvester by lubricants. Moreover, these electromagnetic harvesters have a limited band-width since the magnetic nonlinearity is quite low.
To overcome this damping effect and to have a large bandwidth, we propose the design depicted in Figure 1. Two piezoelectric layers are connected in parallel by a middle layer of steel, having a length L, to reinforce the stiffness of the beam. A center magnet *(M)* is attached to the middle of the beam and inserted between the two fixed magnets *(T)* and *(B)* respectively at the top and the bottom. The three magnets are placed vertically in such a way that all opposed surfaces have the same pole. A wire-wound copper coil is wrapped horizontally around the separation distance between the magnet *(M)* and the two other magnets *(T)* and *(B)*.

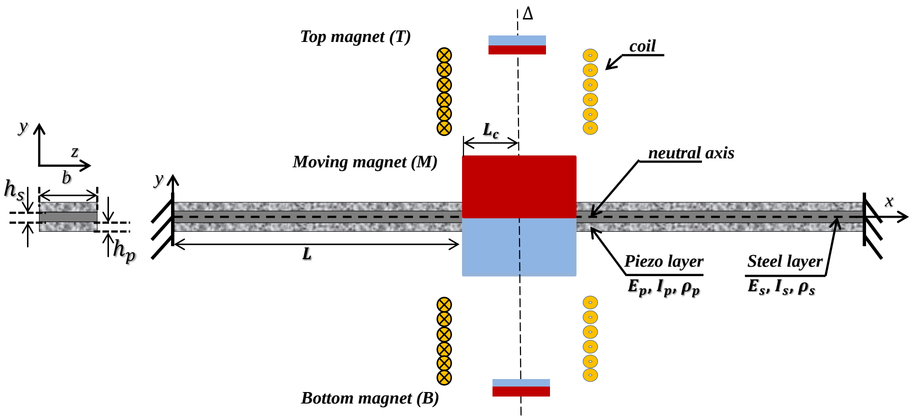

Figure 1: Design of the hybrid piezo-electromagnetic vibrating energy harvester

This axi-symmetric configuration is chosen to ensure that the position of the neutral axis is defined at $y = 0$. The piezoelectric layers are subjected alternatively to tension and compression, when a harmonic excitation $\tilde{Y}(t) = \tilde{Y} sin(\tilde{\Omega}\tilde{t})$ is applied at the device base. The proposed system has a geometric symmetry with respect to the Δ axis as shown in Figure 1. Thus, for simplification reasons, the dynamic study can be limited to a singal symmetric part considering the beam clamped at $\tilde{x} = 0$ and guided at $\tilde{x} = L$ and a rigid body along the length L_c.
The continuum multiphysics problem is derived including the equation of motion of the moving magnet at large displacements and the two transduction equations. A variational approach, based on the extended Hamilton principle is applied to the considered dynamical system as follows:

$$\delta I = \int_{t_1}^{t_2} (\delta \mathcal{L} + \delta \mathcal{W}_{nc})\, d\tilde{t} = 0 \tag{1}$$

where $\mathcal{W}_{nc}$ denotes the work of the non-conservative and external forces and $\mathcal{L}$ is the lagrangien of the system. Doing so, we obtain the following equation of motion,

$$\begin{aligned} & \left[(\rho_s S_s + 2\rho_p S_p)\ddot{\tilde{v}} + (E_s I_s + E_p I_p)\tilde{v}^{(4)} + \tilde{c_m}\dot{\tilde{v}} - N(\tilde{x})\tilde{v}''\right] H_1(\tilde{x}) \\ + & \left[\tilde{F}_m + \frac{\tilde{c_e}}{L_c}\dot{\tilde{v}}\right] H_2(\tilde{x}) = -\left[(\rho_s S_s + 2\rho_p S_p) H_1(\tilde{x}) + \frac{M}{L_c} H_2(\tilde{x})\right]\ddot{\tilde{Y}} \end{aligned} \tag{2}$$

with the associated boundary conditions:

$$\begin{aligned} & \tilde{v}(0,\tilde{t}) = 0 = \tilde{v}'(0,\tilde{t}) \\ & \tilde{v}'(L,\tilde{t}) = 0 = (E_s I_s + E_p I_p)\tilde{v}'''(L,\tilde{t}) - M\ddot{\tilde{v}}(L,\tilde{t}) \end{aligned} \tag{3}$$

where (I_p,E_p, ρ_p, S_p) and (I_s,E_s, ρ_s, S_s) are the second moment of inertia, the Young's modulus, the density and the cross section , respectively, of the two piezoelectric layers and the steel layer. H_1 and H_2 are two heaveside functions defining the elastic part along the length L and the rigid one along L_c. $\tilde{F}_m$ is the magnetic force applied to the moving magnet having M as mass, $\tilde{c_m}$ and $\tilde{c_e}$ are respectively the mechanical and electrical damping. $N(\tilde{x})$ is the nonlinear mechanical term due to the beam mid-plane stretching.
The piezoelectric layers are connected to a resistance R_p. So the current flowing through this resistance is equal to $\frac{V}{R_p}$, where V is the total generated voltage [7]. Therefore, the governing equation for the piezoelectric transduction is:

$$\frac{V}{R_p} = 2b\, e_{31}\int_0^L \frac{\partial\tilde{v}}{\partial\tilde{x}}\frac{\partial\dot{\tilde{v}}}{\partial\tilde{x}}d\tilde{x} - b\, e_{31}(h_s + h_p)\int_0^L \frac{\partial^2\dot{\tilde{v}}}{\partial\tilde{x}^2}d\tilde{x} - \frac{b\, L\epsilon_{33}}{2h_p}\dot{V}(\tilde{x},\tilde{t}) \tag{4}$$

For the electromagnetic transduction, the magnetic field variation in the separation zone gives rise to an induced current $i(\tilde{t})$ respecting Lenz's Law.

$$i(\tilde{t}) = -\left(\frac{\alpha}{R_{load} + R_{int}}\right)\dot{\tilde{v}}|_{\tilde{x}=L} \tag{5}$$

where α is an electromagnetic coupling coefficient expressed as $\alpha = NBl$, where N is the number of turns, B the residual magnetic field and l is the coil length.

3 Solving

For convenience and equation simplicity, the following nondimensional variables are introduced:

$$v = \frac{\tilde{v}}{r}; \quad Y = \frac{\tilde{Y}}{r}; \quad t = \frac{\tilde{t}}{\tau}; \quad x = \frac{\tilde{x}}{L} \quad ; \quad \tau = L^2\sqrt{\frac{\rho_s S_s + 2\rho_p S_p}{E_s I_s + E_p I_p}} \tag{6}$$

Then a reduced-order model is generated by modal decomposition transforming the continum multiphysics problem into a finite degree-of-freedom system consisting of ordinary differential equations in time. Undamped linear bending mode shapes $\phi_i(x)$ of the considered structre are used as basis functions in the Galerkin procedure. Therefore, the deflecion is expressed as $v(x,t) = \sum_{i=1}^n a_i(t)\phi_i(x)$, where $a_i(t)$ is the i^{th} generalized coordinate. The mode shapes $\phi_i(x)$ are normalized such that $\int_0^1 \phi_i\phi_j dx = \delta_{ij}$. The magnetic force is expanded in a Taylor series up to the third order. Assuming that the first mode is the dominant one and considering that the mechanical damping is structural expressed as $c_m = 2\xi\omega_1$, we obtain the following system of three coupled equations representing piezo-electro-mechanical effects in which the motion of the mechanical structure follows a non-linear Duffing equation,

$$\ddot{a_1} + c\dot{a_1} + \omega_1^2 a_1 + \beta_3 a_1^3 = -F_1\ddot{Y} \tag{7}$$

$$\frac{V}{R_p} = I_{elast} - C_p\dot{V} \tag{8}$$

$$i(t) = -\frac{\alpha\, r\,\phi_1(1)}{(R_{load} + R_{int})\tau}\dot{a}_1 \tag{9}$$

and the boundary conditions are:

$$\begin{aligned} & \phi_i(0) = \phi_i'(0) = 0 \\ & \phi_i'(1) = \frac{E_s I_s + E_p I_p}{L^3}\phi_i'''(1) + \frac{M}{\tau^2}\omega_i^2\phi_i(1) = 0 \\ & \phi_i(x) = \phi_i(1)\ \ \forall\ 1 \le x \le 1 + \frac{L_c}{L} \end{aligned} \tag{10}$$

The linear stiffness involves mechanical and magnetic contributions. For this type of harvester, the Duffing term is enhanced by the significant contribution of the nonlinear cubic elastic stiffness. In addition, the overall dissipation is represented by a global coefficient c in Equation (7) which includes a modal and an electrical damping.
The numerical resolution is done by the harmonic balance method coupled with the asymptotic numerical continuation technique [8]. The harvested power is the sum of the two powers transduced piezoelectrically P_p and electromagnetically P_m.

$$P = P_p + P_m = \left(\frac{\omega_1 \frac{2b\, e_{31} r^2}{L\tau} A_1^2 \int_0^1 \phi_1'^2(x)dx}{\sqrt{(2\omega_1 C_p)^2 + \frac{1}{R_p^2}}} \right)^2 \frac{1}{R_p} + R_{load} \left(\frac{\alpha\, r\, \phi_1(1) A_1 \omega_1}{(R_{load} + R_{int})\tau} \right)^2 \tag{11}$$

where A_1 is the frequency response amplitude.

4 Results and discussion

Several numerical simulations have been performed in order to highlight the roles of the elastic guidance and the hybrid transduction in terms of bandwidth and harvested power enhacement. The considered geometric and physical properties of the bimorph beam are given in table 1.

Table 1: Physical and geometric properties of the bimorph cantilever beam

b	Beam width (m)	8 10^{-3}
L	Beam Half-length (m)	50 10^{-3}
L_c	Moving magnet length (m)	10^{-2}
E_s	Effective Steel's Young modulus (GPa)	230.77
E_s^*	Steel's Young modulus (GPa)	210
E_p	Effective piezoelectric's Young modulus (GPa)	69.7
E_p^*	Piezoelectric's Young modulus (GPa)	63
ν_s	Steel's Poisson ratio	0.3
ν_p	Piezoelectric's Poisson ratio	0.31
ρ_s	Steel density (kg/m^3)	7800
ρ_p	Piezoelectric density (kg/m^3)	7500
e_{31}	Piezoelectric coupling coefficient (C/m^2) ou $(N/V.m)$	-13.87
ϵ_{33}	Piezoelectric permittivity (F/m)	1500 ϵ_0
ϵ_0	Vacuum permittivity (F/m)	8.854 10^{-12}

4.1 Performance enhancement by elastic guidance

The case of nonlineaur electromagnetic VEH is investigated in order to demonstrate the imporatnce of the elastic guidance. To do so, the two piezoelectric layers are removed from the device described in Figure 1 reducing it to the steel beam having a thickness $h_s = 0.75\, 10^{-3} m$, the moving magnet (M) and the other two fixed magnets (B) and (T).
In this case, the motion equation of the resulting system can be deduced from the general equation (7) by canceling all piezoelectric terms (ρ_p, E_p, h_p).
Using the magnetic parameters of Table 2, the velocity responses of the system with magnetic levitation and with elastic guidance are depicted in Figure 2. The proposed nonlinear VEH is compared to an electromagnetic harvester based on magnetic levitation [1, 4] having the same magnetic properties for which the maximum power density is approximately $587\, mW$ and the bandwidth is arround $1.9\,\%$. Interestingly, Figure 2 shows that the mechanical nonlinearity due to the beam mid-plane stretching generates large velocities over a wider range of excitation frequencies and consequently an important harvested power. Indeed, the proposed design provides $27\, mW$ in harvested power, $932\, \mu W/cm^3 g$ in power density and $29\,\%$ in bandwidth for a structural damping facteur $\xi = 0.25\%$. The remarkable performances of the proposed device with pure electromagnetic transduction are highlighted in Table 3 with respect to existing magnetic harvesters.

Table 2: magnetic parameters

Magnet type	NdFeB (N35)
Residual magnetic flux density B (T)	1.18
Coercive force (KA/m^3)	860
Magnet density(Kg/m^3)	7400
Dimension of the moving magnet (x,y,z)(m)	(0.01,0.01,0.01)
Coil material	copper
External load resistance R_{load} (Ω)	10^3
Internal resistance R_{int} (Ω)	188
Separation distance r (m)	0.005
Total length of the coil l (m)	0.01

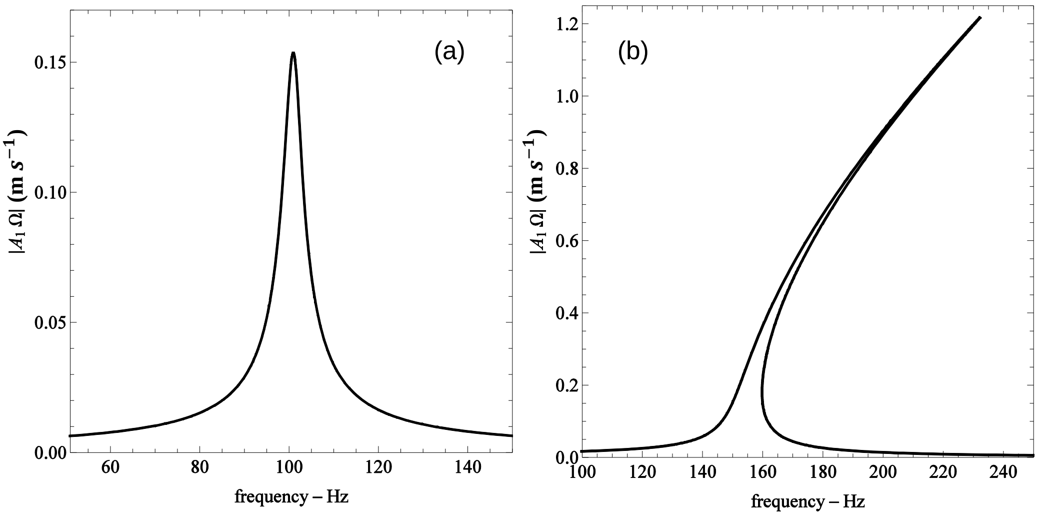

Figure 2: Relative velocity response at excitation amplitudes $F_1\ddot{Y} = 0.9\,g$.(a): case of pure magnetic levitation and (b): case of elastic guidance

Table 3: Characteristics of some energy harvesting designs based on magnetic levitation

Refs	Bandwidth (Hz)	Acceleration (g)	Volume (cm^{-3})	density Pmax per g $(\mu W cm^{-3} g^{-1})$
Sari [9]	4200-5000	50	1.4	0.0057
Xiang [10]	369, 938, 1184	0.76	9.504	0.44
Nguyen [11]	520-580	0.19	0.0271	29.52
Abu Riduan[1]	7-10	0.5	40.18	104.04
Current work	**153-198**	0.9	31.2	**932**

4.2 Performance enhancement by hybrid piezo-electomagnetic transduction

The two transduction thechniques are combined into a hybrid VEH with piezo-magneto-coupling as depicted in figure 1. The VEH is excited at its base by a harmonic excitation $Y(\tilde{t}) = \tilde{Y}\sin(\tilde{\Omega}\,\tilde{t})$ where $\tilde{Y} = 5\,10^{-5}\,m$. The frequency response of the mechanical structure is plotted in Figure 3.

In this configuration, the two transduction techniques are combined into a hybrid VEH and the structural damping has been introduced via a quality factor Q for sandwich materials [12]. The total rigidity of the bimorph beam is amplified which increases the primary resonance frequency of the structure up to $f = 93\,Hz$. A slightly nonlinear frequency response of the considered mechanical structure is plotted numerically in Figure 3, using the data in Tables 1 and 2 as well as the physical and geometrical parameters of Table 4. Remarkably, the elastic part of the harvested energy by the piezoelectric

Table 4: Thickness of the bimorph beam and magnetic intensities of magnets

parameter		**value**
h_s	Thickness of the steel layer (m)	0.3 10^{-3}
h_p	Thickness of the piezoelectric layer (m)	0.16 10^{-3}
Q_T	Magnetic intensity of the magnet T (Am)	5.37
Q_B	Magnetic intensity of the magnet B (Am)	5.42
Q_M	Magnetic intensity of the magnet M (Am)	86

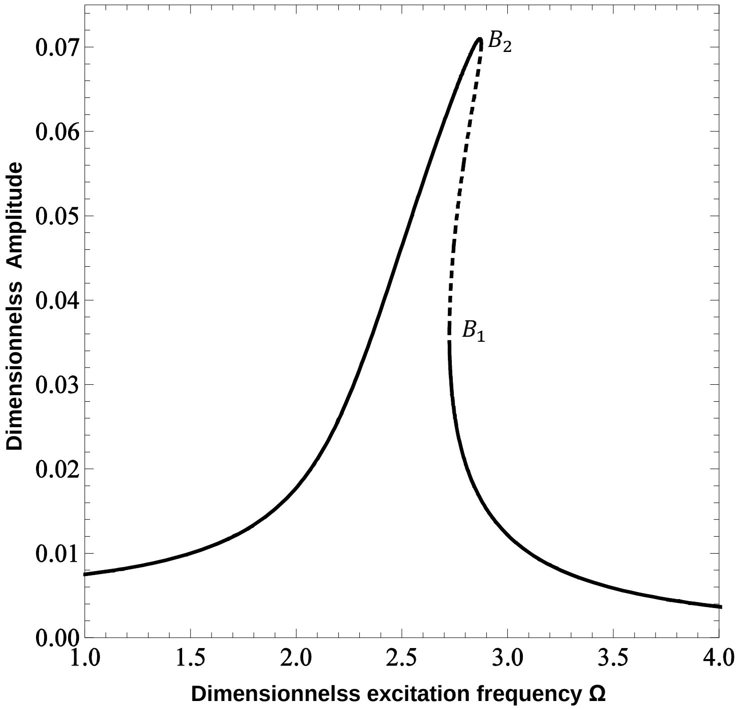

Figure 3: Numerical frequency response of the hybrid VEH with the exception of the quality factor $Q = 50$. Solid line denotes stable periodic solutions and a dashed line represents unstable periodic solutions.

transduction is nil when the beam vibrates linearly. Therefore, it is necessary to reach large displacements to take advantage of the PZT layers. In practice, the external excitation at the device base must be significantly high. Numerical simulations show that the device bandwidth and the density power can reach $10\,\%$ and $1190\,\mu W/cm^3 g$ respectively.

5 Conclusion

The non-linear dynamics of a hybrid piezo-electromagnetic vibrating energy harvester (HVEH) was modeled including the main sources of non-linearities. The continuum mutiphysics problem was derived thanks to the extended Hamilton principle. The modal Galerkin decomposition method was used in order to obtain a reduced order model consisting of a nonlinear Duffing equation of motion coupled with two transduction equations. The resulting system was solved using the harmonic balance method coupled with the asymptotic numerical continuation technique.
Several numerical simulations have been performed to highlight the performance of the proposed HVEH. Particularly, the power density and the bandwidth can be boosted up to $60\,\%$ and $29\,\%$ respectively compared to the case of a VEH with a pure magnetic levitation thanks to the nonlinear elastic guidance. Moreover, the hybrid transduction permits the enhancement of the power density up to $100\,\%$ ($44\,\%$ transduced piezo-electrically and $56\,\%$ harvested magnetically).
Future work will include the optimization of the performances of the proposed nonlinear HVEH when the piezoelectric layers are replaced by PZT patches close to the clamped extremities of the beam.

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