ENOC 2014, July 6-11, 2014, Vienna, Austria

Energy transfer in externally driven periodic nonlinear structures

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Summary. The collective dynamics of an externally driven periodic lattice of coupled nonlinear Duffing-Van Der Pol oscillators is modeled and investigated under primary resonance. The considered lattice has the particularity of including linear and nonlinear coupling between each two adjacent oscillators. The analytico-computational model is based on the method of multiple scales coupled with standing wave modal decomposition transforming the nonlinear differential system into a set of coupled complex algebraic equations which are numerically solved. Several numerical simulations have been performed and particularly for the case of two coupled oscillators, remarkable energy and bifurcation topology transfers between the lattice elements are identified.

Introduction

Nonlinearities play an increasingly dominant role in dynamics, especially when it comes to functionalize them, particularly, in periodic structures in order to control their collective nonlinear dynamics which is regaining attention in several physical domains. For instance, in the field of acoustics, Manktelow et al. [1] focused on the interaction of wave propagation in a cubically nonlinear mono-atomic chain and Marathe et al. [2] studied wave attenuation in nonlinear periodic structures while, in Optics, Heinrich et al. [3] investigated the collective nonlinear dynamics in arrays of coupled optomechanical cells. Moreover, in micro and nanotechnology, dynamic behavior investigations of an array of N nonlinearly coupled microbeams have been performed by Gutschmidt et al. [4] using a continuum model, and Lifshitz et al. [5], using a discrete model. In this context, we investigate a particular model for the collective nonlinear dynamics of periodic lattices. It consists of an array of coupled Duffing-Van Der Pol oscillators under primary resonance. The main goal is to understand how coupled nonlinearities influence external driven operation and how they may be used to enhance and control bifurcation topologies and energy transfers.

Model and results

The proposed model involves a finite degree of linearly and nonlinearly coupled Duffing-Van Der Pol Oscillators externally driven by a sinusoidal force, as shown in Figure 1. The corresponding set of coupled equations of motion EOM can be written in the following form:

$$m\ddot{u}_n + K_d(-u_{n+1} + 2u_n - u_{n-1}) + c\dot{u}_n + Ku_n + \mu(u_n - u_{n+1})^3 + \mu(u_n - u_{n-1})^3 + \beta u_n^2 \dot{u}_n = F\cos(w_D t + \Phi) \quad (1)$$

where $u_n(t)$ describes the deviation of the n^{th} resonator from its equilibrium, with $n = 1 \cdots N$ and fixed n boundary conditions $u_0 = u_{N+1} = 0$, m is its effective mass, $K = mw_0^2$ is its effective spring constant, with w_0 the resonant frequency, μ is the cubic spring constant, or Duffing parameter, $c = 2maw_0$ is the linear damping (a is the damping factor), $K_d = \gamma K$ is the coupling spring constant, and β is the coefficient of the Van Der Pol damping. The term proportional to F on the right-hand side is the standard direct drive, possibly shifted by a phase Φ, and its frequency is set an amount $\varepsilon\Omega_D$ away from the resonant frequency w_0. Ω_D is the detuning parameter and ε is a small nondimensional bookkeeping parameter.

To treat the set of coupled equations of motion (1) analytically, we use secular perturbation theory combined with a multiple scales analysis, where we vanish N secular terms that act to drive the oscillators at their resonance frequencies. Hence, to extract the equation for the m^{th} amplitude (m), we make use of the orthogonality of the standing wave modes. Since we are interested in the steady state motion, the N differential equations have been transformed into a set of N coupled nonlinear complex algebraic equations for the time independent mode. By solving numerically the resulting system, the overall collective response of the array can be plotted with respect to the detuning parameter Ω_D. The solutions for the response intensity of two oscillators as a function of frequency are shown in Figure 2. With two resonators there are regions in frequency where three stable solutions can exist. If all the stable solution branches are accessible experimentally then the observed effects of hysteresis might be more complex than in the simple case of a single resonator. This is demonstrated in Figure 3, where we plot the time and space averages of the square of the resonator displacements $I = \frac{1}{2}\sum_{n=1}^{2} < x_n^2 >$, where the angular brackets denote time average.

Conclusion

A discrete analytico-computational model has been developed in order to investigate the collective nonlinear dynamics of specific periodic lattices. In practice, it can be used to design innovative vibrating energy harvesters which are based on an array of coupled levitated magnets.

Acknowledgments

This project has been performed in cooperation with the Labex ACTION program (contract ANR-11-LABX-01-01).

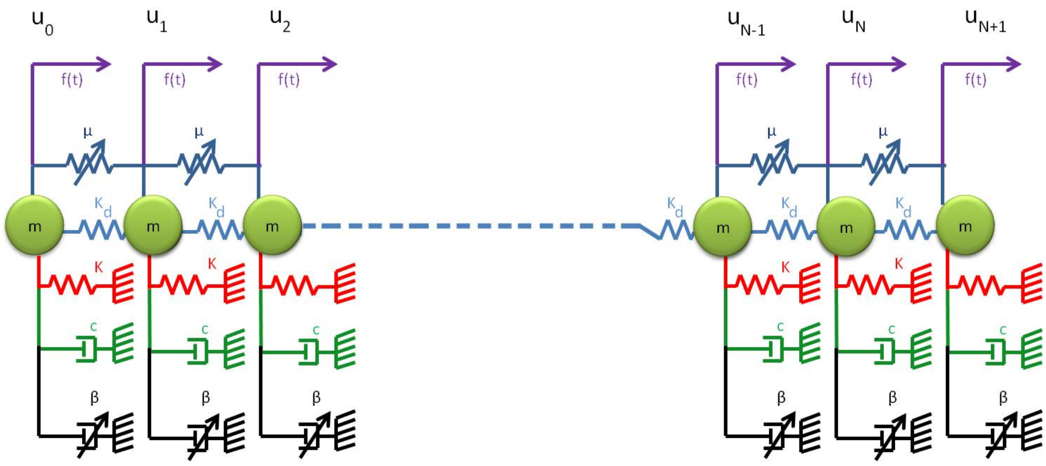

Figure 1: Coupled Duffing-Van Der Pol mass-spring chain, with linear damping

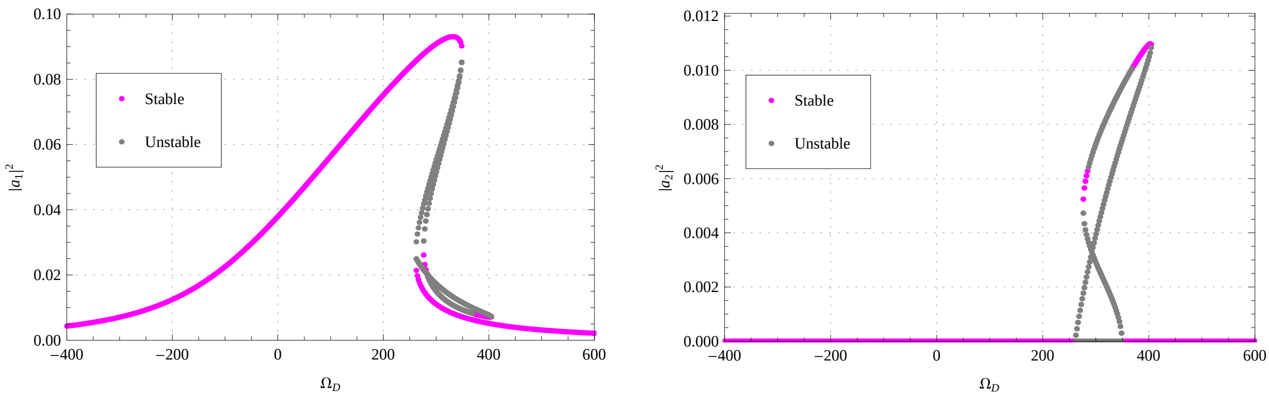

Figure 2: Response intensity of two resonators as a function of the detuning parameter Ω_D, for a particular choice of the equation parameters. The left curve shows the first oscillator, and the right one shows the second, with magenta curves indicating stable solutions and gray curves indicating unstable solutions.

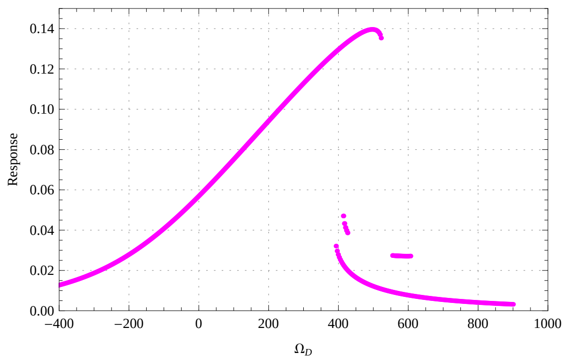

Figure 3: The average response intensity, where these branches correspond to stable solutions in Figure 2.

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