Basins of attraction of coupled nonlinear resonators in periodic lattices

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Summary
Collective dynamics in periodic lattices of coupled nonlinear Duffing-Van Der Pol oscillators is modeled and investigated under simultaneous external and parametric resonances. The resonators are coupled with linear and nonlinear springs. Numerical simulations have been performed in the case of two coupled oscillators for which the basins of attraction have been analyzed in the multistability domain in order to check the efficiency of the multimode branches.

Introduction

Interest in the nonlinear dynamics of periodic nonlinear lattices has grown rapidly over the last few years. Actually, it exists a practical need to understand nonlinearities and functionalize them in order to efficiently exploit the collective nonlinear dynamics of smart structures. For instance, Lifshitz et al. [1] investigated the dynamic behavior of an array of N coupled micro-beams using a discrete model. Manktelow et al. [2] focused on the interaction of wave propagation in a cubically nonlinear mono-atomic chain, while Bitar et al. [3] investigated the bifurcation and enery transfers in periodic lattices of coupled nonlinear Duffing-Van Der Pol oscillators under an external excitation. Particularly, the basins of attraction can be used for qualitative as well as quantitative analysis of the collective dynamics robustness. In a nonlinear nanomechanical resonator, Kozinsky et al. [4] experimentally probe the basins of attraction of two fixed points. Moreover, Sliwa et al. [5] investigated the basins of attraction of two coupled Kerr oscillators. Furthermore, Ruzziconi et al. [6] studied frequency response curves, behavior charts and attractor-basins phase portraits of a considered NEMS constituted by an electrically actuated carbon nanotube. In this context, an array of coupled Duffing-Van Der Pol oscillators under simultaneous primary and parametric resonances is developed and the distribution of the basins of attractions is analyzed for multistable solutions including single and multi-modes branches.

Model and basin of attraction analysis

The proposed model presents an array of a finite coupled Duffing-Van Der Pol oscillators, under simultaneous external and parametric excitations. The scaled equation of motion (EOM) governing the behavior of the n^{th} resonator can be written as:

$$\ddot{u}_n + \varepsilon\frac{\omega_0}{Q}\dot{u}_n + \omega_0^2 u_n + \varepsilon h\cos[2(\omega_0+\varepsilon\Omega)t]u_n + \frac{1}{2}\varepsilon d(-u_{n+1}+2u_n-u_{n-1})$$
$$+\alpha[(u_n-u_{n+1})^3+(u_n-u_{n-1})^3] + \eta u_n^2\dot{u}_n = \varepsilon^{\frac{3}{2}} g\cos[(\omega_0+\varepsilon\Omega)t], \quad (1)$$

where u_n is the displacement of the n^{th} oscillator, with fixed boundary conditions $u_0 = u_{N+1} = 0$, ω_0 and Ω are respectively the natural frequency and the detuning parameter. Q is the quality fractor, d represents the linear coupling, α is the cubic spring constant and η represents the Van Der Pol damping coefficient, h and g are parametric and external excitation amplitudes respectively, and ε is a small dimensionless parameter.

The method of multiple time scales was used to solve the coupled EOM analytically. With the expectation that the motion of the resonator far from its equilibrium will be on the order of $\varepsilon^{\frac{1}{2}}$, we try a solution of the form:

$$u_n(t) = \varepsilon^{\frac{1}{2}}\sum_{m=1}^{N}(A_m(t)\sin(\frac{nm\pi}{N+1})e^{i\omega_0 t} + c.c.) + \varepsilon^{\frac{3}{2}}u_n^{(1)}(t) + \cdots \qquad n = 1,...,N, \quad (2)$$

where $T = \varepsilon t$ is a slow time variable. The slowly varying amplitudes $A_m(T) = (a_m(T) + ib_m(T))e^{i\omega_0 t}$ obeys to $2N$ differential equations. In Figure 1, we display the response amplitude of the first oscillator, in function of the detuning parameter of two coupled oscillators, for specific design parameters. Remarkably, there are frequency bands where four stable solutions can exist. The multivaluedness of the response curves due to the nonlinearity has a significance from the physical point of view because it leads to jump phenomena wich are localized at the bifurcation points.

The basins of attraction are numerically plotted to investigate the trajectories of the system response and the probability for which the system follows either the resonant or non-resonant, for single or double mode branches. Although, they are usually plotted in the phase plane $(u_n, \dot{u}_n)$, we chose to represent them in the Nyquist plane. Several numerical integrations of differential equations have been performed for a specific domain of initial conditions, in order to localize the maximum amplitude $|A_1|^2$ in the steady-state domain. As shown in Figure 1, $|A_1|^2$ takes one of four values, depending in the chosen initial conditions which allows for the representation of the corresponding basins of attraction.

Conclusion

The collective nonlinear dynamics of periodic nonlinear lattices was modeled for specific discrete systems of coupled Duffing-VDP oscillators under simultaneous primary and parametric excitations. The case of two coupled oscillators was investigated for a specific design parameters for which, the basins of attraction have been analyzed in the multistability domain for two coupled nonlinear oscillators to quantitatively asses the efficiency and reliabiliy of additional branches when used in energy harvesting applications.

Acknowledgments
This project has been performed in cooperation with the Labex ACTION program (contract ANR-11-LABX-01-01).

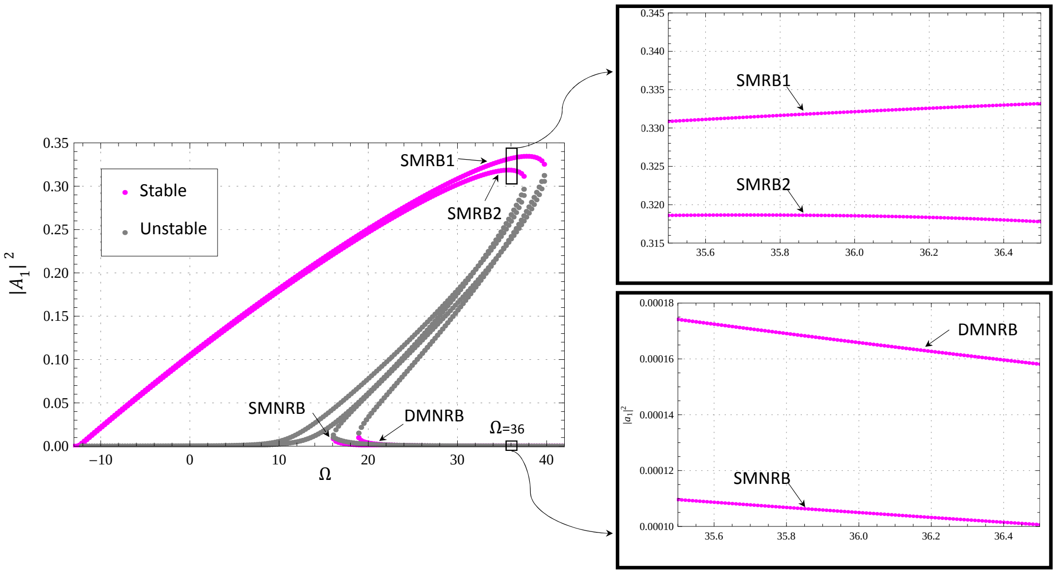
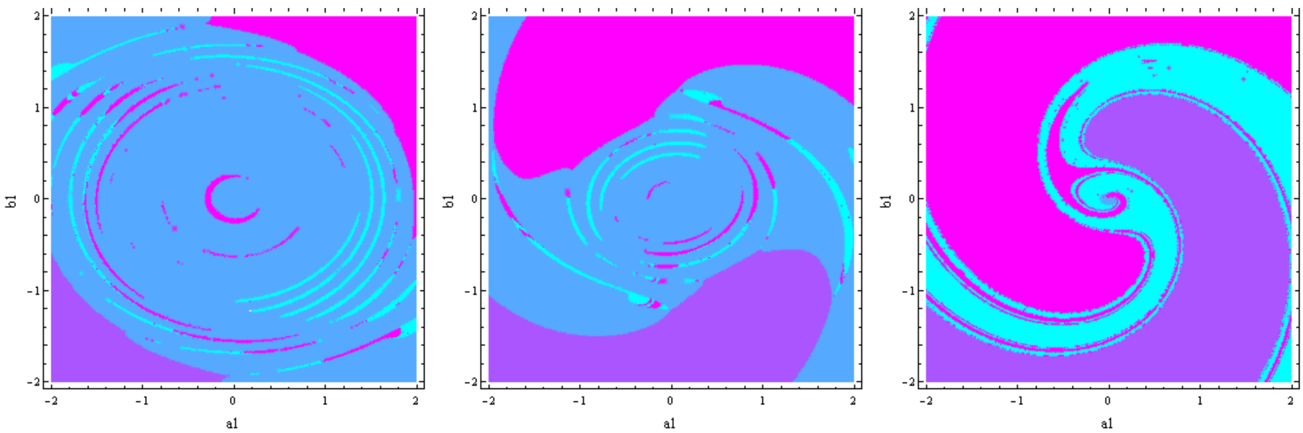


Figure 1: Response intensity as a function of the detuning parameter Ω for the first oscillator, where SMRB1 and SMRB2 are **S**ingle **M**ode **R**esonant **B**ranches due respectively to primary and parametric resonances, SMNRB and DMNRB are respectively **S**ingle and **D**ouble **M**ode **N**on **R**esonant **B**ranches.



(a) a2=2 (b) a2=1 (c) a2=0

Figure 2: Variation of the basins of attraction in the Nyquist plane $(a1, b1)$, with respect to the inital condition $a2$ for fixed detuning parameter $\Omega = 36$ and $b2 = 0$. Magenta, purple, blue and cyan colors indicate respectively SMRB1, SMRB2, SMNRB and DMNRB.

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