Collective dynamics of coupled nonlinear pendulums under simultaneous external and parametric excitations

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Summary. In order to investigate the collective nonlinear dynamics of an array of coupled pendulums under simultaneous external and parametric excitations, a computational model is developed while considering the main sources of nonlinearities. The equations of motion are solved using the harmonic balance method (HBM) coupled with the asymptotic numerical method (ANM). Numerical simulations are performed in the case of two coupled pendulums in order to investigate the complexity of the frequency responses in terms of bifurcation topologies and energy transfer.

Introduction

The sine-Gordon model and its discrete analog have attracted interest of people working in quite different fields. For instance, the collective dynamical behavior of an array of coupled pendulums with a small fraction of random long-range connection has been investigated under external excitation [1, 2]. Gavielides et al. [3] have investigated the effect of impurity introduced into a lattice and their ability to control the dynamic behavior of an array of coupled nonlinear chaotic oscillators, while Thakur et al. [4] have examined a pendulum array with harmonic coupling and horizontal sinusoidal driving. Hai-Qing et al. [5] and Alexeeva et al. [6] demonstrated the stabilizing effect of adding mass and length impurities on a chain of pendulums parametrically excited. In this context, a computational model for the nonlinear dynamics of a chain of coupled pendulums under simultaneous external and parametric excitations is developed. The principal goal is to track the collective dynamics of the considered system in terms of bifurcation topologies and energy transfer with respect to the excitation amplitudes.

Design and model

The considered system is depicted in Figure 1. It is composed of an horizontal axle A, of total length l, suspended at its ends by "frictionless" bearings. Along this axle, at equally spaced intervals, there are N equal pendulums. Each pendulum consists of a rigid rod, attached perpendicularly to the axle, with a mass at the end. At rest, all the pendulums point down the vertical, a is the distance of the center of mass from the central axis, g the gravity acceleration, θ_i is the angle between the i^th pendulum and the downward vertical, k_2 is the torque constant and k_4 is the coupling nonlinear stiffness. The pendulums have the same moment of inertia $I = ml^2$. The considered system is excited by two excitation forces at the drive frequency ω_e. The first one is an external force $F\cos(\omega_e t)$ applied on one or several pendulums, and a parametric excitation $4A_e{\omega_e}^2\cos(2\omega_e t)$ due to the base excitation of the system. Hence, the equation of motion of the n^{th} pendulum can be written as:

$$\begin{aligned} & ml^2\frac{d^2\theta_n}{dt^2} + \alpha l\frac{d\theta_n}{dt} + k_2\left(2\theta_n - \theta_{n+1} - \theta_{n-1}\right) + k_4\left(\left(\theta_n - \theta_{n+1}\right)^3 + \left(\theta_n - \theta_{n-1}\right)^3\right) \\ & +ml\left[g + 4A_e\omega_e^2\cos\left(2\omega_e t\right)\right]\sin\left(\theta_n\right) = Fcos(\omega_e t) \end{aligned} \tag{1}$$

The boundary conditions associated to Equation (1) are $\theta_0 = 0$ and $\theta_{N+1} = 0$. First, $\sin(\theta_n)$ is expanded in Taylor series up to the third order. Then, the resulting equation is numerically solved in the frequency domain using the harmonic balance method coupled with the asymptotic numerical continuation technique.

Numerical simulations

In order to investigate the effect of adding an external force, we plot numerically the frequency responses for an array of two coupled pendulums. In the case of a pure parametric excitation ($F = 0$), the two curves of θ_1 and θ_2 are identical due to the symmetry of the equations, as shown in Figure 2(a). Moreover, we notice that the resonant stable solution is limited to the frequency range $[10.29\ 10.77]\,rad/s$ while the trivial solution $\theta_n = 0$ is anywhere else. Figure 2(b) displays the frequency responses of the two pendulums under simultaneous external and parametric excitations. Remarkably, adding a

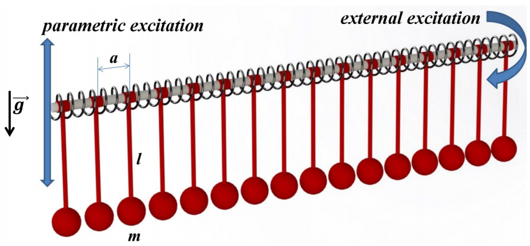
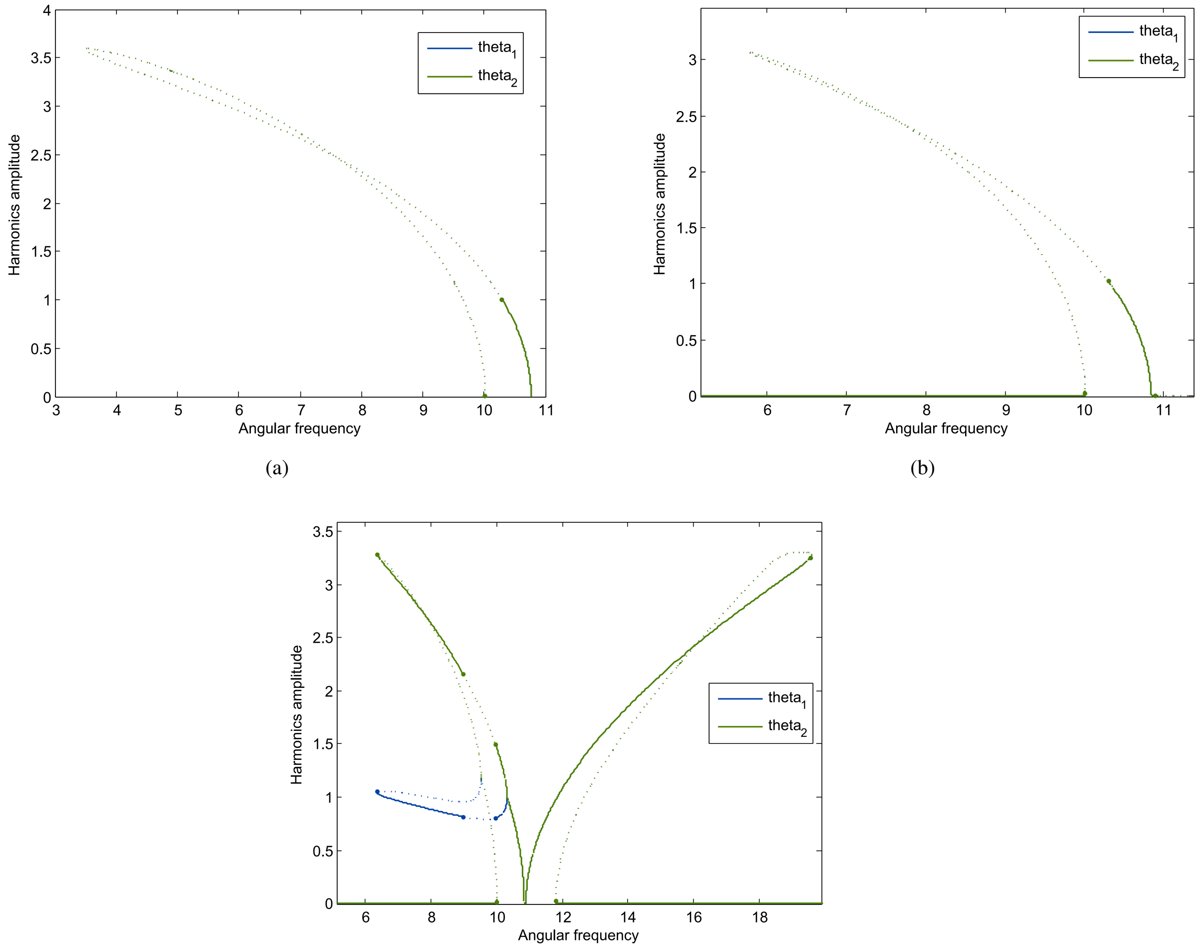


Figure 1: An array of coupled pendulums under simultaneous parametric and external excitations.



(a) (b)

(c)

Figure 2: Forced frequency responses of the two coupled pendulums for the following set of parameters: $m = 0.1$, $l = 0.1$, $g = 9.81$, $A = 0.004$, $k_2 = 0.01$, $k_4 = 0.005$, $\alpha = 0.001$, $F = 10^{-6}$. (a): under pure parametric excitation ($F = 0$), (b): under simultaneous parametric and external excitations, and (c): the two pendulums are excited with the same parametric force while only the first pendulum is externally forced. Solid curves indicate stable solutions and dashed curves indicate unstable solutions.

small external force has an impact only on the unstable solution. Finally, we perform numerical simulations for the case of simultaneous excitations while the external one is localized on the first pendulum. Since the symmetry of the problem is broken, there are large differences between the two frequency responses as shown in Figure 2(c). For $\omega_e > 10.85\, rad/s$, a new branch is added providing a non-zero solution and compared to the two first cases, we can see the existence of new stable branches for $\omega_e < 10.85\, rad/s$.

Conclusions

A computational model for the nonlinear dynamics of an array of coupled pendulums under simultaneous parametric and external excitations has been developed. Particularly, it is shown that we can take advantage of the external force to stabilize the structure and increase the performances of the resulting collective dynamics.

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