

An unsupervised pattern recognition approach for AE data originating from fatigue tests on polymer-composite materials*

D. D. Doan, E. Ramasso, V. Placet, L. Boubakar, N. Zerhouni

Abstract— Acoustic Emission (AE) technique is gaining more and more interest for structural health monitoring (SHM) in polymer-composite materials. Recent literature has shown that using appropriate pattern recognition techniques (PRT), the identification of the natural clusters of acoustic emission data can be obtained. Despite these recent and valuable advances and to achieve health assessment of composite materials, the scientific community faces two major challenges: (i) develop real-time approaches and (ii) propose clustering approaches able to process in in-service-like situation, i.e. in case of high AE activity generated simultaneously from many damage sources in material, from damage progression and cumulated damage and from noise.

This work investigates acoustic emission generated during tension fatigue tests carried out on a carbon fiber reinforced polymer (CFRP) composite specimen. Since fatigue data processing, especially noise reduction remains a big challenge in AE data analysis; a simple null-value filtering and a noise modeling have been proposed in the present work to tackle this problem. A Davies-Bouldin-index-based progressive feature selection has been implemented to reduce high dimensional fatigue dataset. A classifier offline-learned from quasi-static data is then used to classify the processed data to different AE sources. An adaptation has been studied to enable the classifier to generate new class, i.e. AE source, for unidentified AE events. With efficient proposed noise removal and automatic separation of AE events, the results of this work provide an insight into fatigue damage development in composites and then ability to health assessment which is necessary for residual life prediction.

I. INTRODUCTION

AE testing has become a recognized nondestructive test (NDT) method, commonly used to detect and locate faults in mechanically loaded structures and components. AE could provide comprehensive information on the origination of a discontinuity (flaw) in a stressed component and also provide information pertaining to the development of this flaw as the component is subjected to continuous or repetitive stress. Moreover, the method has been developed and applied in

numerous structural components, such as steam pipes and pressure vessels, and in the research areas of rocks, composite materials, and metals.

Acoustic emissions (AEs) are the stress waves produced by the sudden internal stress redistribution of the materials caused by the changes in the internal structure [1]. Possible causes of the internal-structure changes are crack initiation and growth, crack opening and closure, dislocation movement, twinning, and phase transformation in monolithic materials and fiber breakage and fiber-matrix debonding in composites. Most of the sources of AEs are damage-related; thus, the detection and monitoring of these emissions are commonly used to predict material failure.

With a huge noisy amount of data originating from fatigue loading tests, a major challenge in the use of AE technique is to associate each signal to a specific AE source related to noise or a damage mechanism. Consequently, AE signals recorded during tests must be segmented into clusters based on similarity measures. However, this analysis is a non-trivial task for two main reasons. First, AE signals are complex objects that must be characterized by multiple pertinent features. Second, there is no a priori knowledge of the acoustic signatures of damage events and these are assumed rather scattered.

In the literature, dealing with the challenge of big data due to high sensitivity of AE sensors and to long-term fatigue loading experiments, many processing approaches have been proposed by [2], [3], [4] and [5]. In [2] and [3], it is considered that only signals with amplitude higher than 70 dB or recorded above 80% of peak load contain information related to damage mechanisms. This filtering is subjectively supposed to be efficient in terms of quantitative reduction but it could take a serious risk at missing low and medium energy AE sources that condition the onset of more severe damage mode. In [4], ‘friction emission’ tests in which the maximum cyclic load was decreased to a level that was insufficient to generate crack growth were performed to understand the AE signal characteristics arising from hydraulics, machine start and stop, slippage, grating between fracture surfaces (also referred to as ‘fretting’), and abrasion of load train. All of the AE events at this lower peak load were therefore assumed to be due to friction emission. Emission having the characteristics of friction emission was then filtered. Friction emission testing was useful and did provide reference waveforms to aid in the differentiation of noise from cracking. However, it did not provide all-inclusive reference parameters for data filtering. This is because the loads were lower than those in the formal fatigue tests. Besides, this specialized kind of test requires a specific load level mentioned above that is not always obviously determined. A

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more complex denoising process developed by [5] that combines PCA and k-means and several validation techniques was presented to be able to classify more than 60% of the detected signals as noise, before the application of a SOM algorithm to separate AE events from the residual noise in the remaining dataset recorded during long time corrosion monitoring of a pre-damaged post tensioned concrete beam. In spite of the capacity of noise removal, its implementation induces high computational complexity.

High dimensional feature space reduction is still a remaining challenge to statistic processing and classification of AE data. In the literature, many approaches for AE data processing [6]–[8] are conditioned by Principal Component Analysis (PCA). The latter provides an automatic feature space reduction as well as extraction of relevant components subset from the original features set. This algorithm assumes that 1) the linear combination of features improves the relevancy of the principal components and 2) a large variance implies meaningfulness. Other approaches [9]–[11] rely on a specific subset of features. Ones select à priori conventional feature subset as energy, rise time, duration, amplitude, and so on [9] or reduce feature dimension space by using complete link hierarchical clustering in order to merge the correlated features into groups [10]. Those apply a greedy approach that generates all possible feature combinations and then selects the one which optimizes a given criterion [11], [12]. The goal of the criterion is generally to evaluate the quality of the partition provided by the clustering. Most of criterions are based on the Euclidean distance to assess the membership of an AE hit to a given cluster. However, the applicability of this approach is limited to clustering algorithms which are based on the Euclidean distance. The PCA is generally used jointly with the K-means [13]. The main reason to account for the performance of this couple is actually due to the link between both tools. Compared to usual approaches based on K-means or FCM [14] that use the Euclidean distance, the ARI-based GK algorithm proposed by [15] takes the distribution of the data points into account with a modified Mahalanobis distance for each cluster which is iteratively adapted to fit ellipse-shaped clusters. Low density and high scattering nature of AE data makes using ellipses more appropriate than circles to represent AE data. In the GK algorithm, the covariance between each pair of features is estimated so that possible redundancy or complementarity between features can be taken into account.

The main objective of this work is unsupervised damage detection using clustering algorithms, where each cluster is supposed to represent a specific AE source related or not to a damage family. Accurate damage detection is a difficult problem involving several challenges [16]:

Challenge 1 *The choice of features.* According to the algorithm used for damage detection, different subsets of features may lead to different results.

Challenge 2 *The number of damage families* is not always well defined and well known.

Challenge 3 *Robustness of algorithms to initialization* of clustering algorithms has to be ensured for practical real-life applications in order to retrieve results easily.

Challenge 4 *The revision of models* obtained by clustering without re-training (using all past data but only the current ones).

Challenge 5 *Cluster labeling* requires others NDT as validation measures of cluster analysis.

In this paper, we propose a methodology which deals with the challenges 1-2-3-4. Moreover, a noise removal tool has been implemented to overcome the problems related to computing approaches involving time consuming, computational cost and accuracy gain. A visualization of the complete method is shown in Fig. 1 as a flowchart diagram.

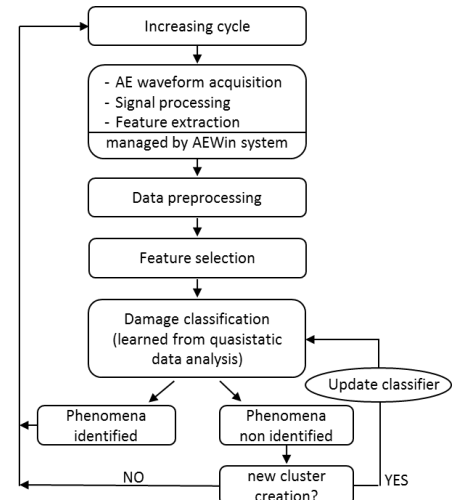


Figure 1. Unsupervised damage detection methodology

II. UNSUPERVISED PATTERN RECOGNITION METHODOLOGY

A. AE FATIGUE DATA PREPROCESSING

Eliminating extraneous background noise remains a big challenge to AE investigations in fatigue. Background noise is particularly serious in fatigue for two reasons. The AE signal level in fatigue is relatively low, while the cyclic-loading process is inherently noisy. Sources of background noise in electro-hydraulic test machines, such as that used in this investigation, are of four types. Electrical noise on the system usually is of amplitude of about 20 dB. Noise emanating from servo-valves and hydraulic pumps can reach a significant signal level. Noise issues from relative movement in the load train. Under conditions of reversed cyclic loading this signal level can become very high. With the reversal of loading (stress ratio $R < 0$), mechanical fretting noise increases in the specimen grips as the compression force also increases. The latter type of noise is the hardest to eliminate. Its characteristics are very similar to those of the acoustic emission from cracks. More details about identifying this kind of noise will be discussed in the section III.

a) Signal screening

Continuous background noise due to hydraulic flows is essentially eliminated from the AE signal by a floating signal threshold, which is automatically adjusted at a 40 dB level.

b) Data cleansing

This process deals with detecting and removing errors and inconsistencies from data in order to improve the quality of

data. Indeed, due to sensitivity and complexity of acquisition devices as well as inaccessibility to code sources of feature extraction process, accurate and consistent data are hardly provided by commercial AE waveform acquisition systems. Thus selectively choosing data based on knowledge gained by individuals performing the measurement is required to carry out in favor of feature selection process.

c) *Noise-model-based filtering*

As mentioned above, such a noise model proposed by [4] is expected to be able to filter out signals having the characteristics of friction emission. However, this approach depends on a strong assumption that there are no damage phenomena taking place under the level determined by experience. To guarantee the mere presence of noise such as mechanical rubbing, electromagnetic interference (EMI), in this work, only AE signals recorded during setting-in-place time, i.e. before loading, are considered for noise modeling. This model is then used to filter out AE events during the test which have the same characteristics as the modeled noise.

B. PROGRESSIVE SELECTION ALGORITHM OF AE FEATURES

The goal of this section is to propose an automated technique to detect relevant feature subsets for clustering of AE events. In contrast to feature reduction procedures (e.g. based on correlation dendrograms in [6]) or exhaustive search of global optimal feature combinations in [11], the principle of the presented approach is to combine progressively each feature from an available feature space with an initial feature subset.

Our feature selection is then realized by minimizing value of Davies and Bouldin (DB) index [17]. It is calculated by the following formula:

$$DB = \frac{1}{k} \sum_{i=1}^k \max_{i \neq j} \left\{ \frac{d_i + d_j}{D_{ij}} \right\}$$

where d_i and d_j are the average within-class distances of clusters i and j respectively, and D_{ij} denotes the distance between the two clusters i and j .

This clustering validity index has been used by several authors in order to select optimal cluster number [10] or to evaluate feature subset partition [11]. Due to the way it is defined, as a function of the ratio of the within cluster scatter, to the between cluster separation, a lower value of this criterion performs a good compactness and a good separation of dataset partition.

The figure 2 shows the diagram of the proposed algorithm. Considering an initial selected feature set denoted S (empty by default), the algorithm will take each of available features from F to create a new subset with S . This subset is then partitioned by a clustering algorithm proposed by [15]. In fact, one of big issues on partition calculated by two well-known clustering algorithms, namely K-means and FCM, is that their result may vary importantly between several tests, in particular due to parameters' initialization and to the noise present in the data. Therefore, [15] proposed a method based on the adjusted rand index - ARI [18] to quantify the repeatability of the clustering results. This measure is then

exploited to initialize a Gustafson-Kessel (GK) algorithm [19]. This enhanced clustering algorithm is flexible when detecting particular shape of data points and is able to provide repeatable partition.

Subset partition calculated by this ARI-based GK algorithm is then evaluated by DB criterion. The additional feature whose subset minimizes the value of DB index is selected as relevant one. Thus, this feature will be removed from F to S . In each iteration, the procedure generates k new subsets if the number of features remaining in F is k , because each new subset contains the features from S plus a new one taken from the remaining ones in F . The algorithm stops when no new subsets can improve the DB criterion. For each iteration i , we calculate improvement rate by the following formula:

$$IR(i) = \frac{DB(S_i) - DB(S_{i-1})}{DB(S_{i-1})}$$

where $IR(i)$ is improvement rate in the i^{th} iteration, $DB(S_i)$ and $DB(S_{i-1})$ are value of DB index of the best feature selection for the i^{th} and $(i-1)^{th}$ iteration. The sign of IR indicates if the DB criterion is improved (negative) or not (positive).

In the last iteration j , i.e. $IR(j) > 0$, if $IR(j) < \min_{i \neq j} |IR(i)|$ then the best-DB-index feature can be added to S to establish the final selected feature set.

Figure 3 illustrates an example of this implementation. Given an available feature set F of 5 elements $\{1, 2, 3, 4, 5\}$ and an empty selected feature set S . For the first iteration, the algorithm determines 2 as the feature which gives the best score of DB index. Therefore, this feature is removed from F to S . In the second iteration, each remaining feature in F is combined with the previous selected one to constitute 4 subsets of 2 elements. The selected feature is 5 because the partition is more improved due to the combination between the features 2 and 5. Indeed, the best value of DB index passes from 0.1 in the first iteration to 0.05 in the 2nd iteration, which means an improvement of 50%. In contrast, the positive value of IR in the 3rd iteration represents a degradation of 2% in terms of final partition. Thus, the iteration is stopped. However, this tiny increase is tolerable. In fact the two subsets $\{2,5\}$ and $\{2,5,4\}$ have the similar score, it's means that the addition of the feature 4 would not change the final performance of partition.

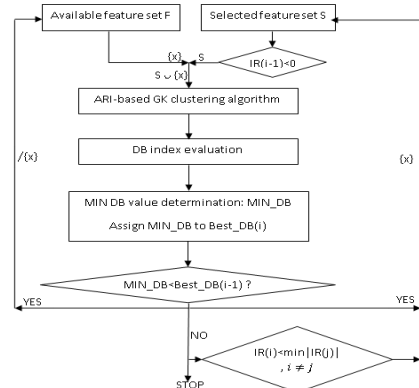


Figure 2. Progressive feature selection diagram

1 st iteration	2 nd iteration	3 rd iteration
S= \emptyset ; F={1,2,3,4,5}; Best_DB(0)=100;	S={2}; F={1,3,4,5}; Best_DB(1)=0.1;	S={2,5}; F={1,3,4}; Best_DB(2)=0.05;
Subset generation		
{1}, {2}, {3}, {4}, {5}	{2,1}, {2,3}, {2,4}, {2,5}	{2,5,1}, {2,5,3}, {2,5,4}
Apply ARI-based GK clustering algorithm to each subset and evaluate its DB index		
DB ₁₁ = 0.2 DB ₁₂ = 0.1 DB ₁₃ = 0.5 DB ₁₄ = 0.9 DB ₁₅ = 0.3	DB ₂₁ = 0.12 DB ₂₃ = 0.1 DB ₂₄ = 0.08 DB ₂₅ = 0.05	DB ₃₁ = 0.4 DB ₃₃ = 0.14 DB ₃₄ = 0.051
Feature selection		
MIN_DB= 0.1 Best_DB(1)=0.1 IR(1)= -0.999 S=S \square {2} F=F\{2}	MIN_DB= 0.05 Best_DB(2)=0.05 IR(2)= -0.5 S=S \square {5} F=F\{5}	MIN_DB= 0.051 Best_DB(3)=0.051 IR(3)= 0.02 => STOP S=S \square {4} F=F\{4}

Figure 3. : Example of selection algorithm

C. AE SOURCE CLASSIFICATION

Tensile tests were performed in order to generate three main families of damage related to matrix, interface and fibers. However, it is very difficult to carry out cluster analysis on a large quantity of signals originated from fatigue loading test. Otherwise, quasi-static tensile tests from which a smaller number of AE events are detected could perform the same damage modes as the nature of composite specimens subjected to different loading tests is similar. Thus in the first stage, the feature selection algorithm presented in (B) was used to generate a subset of relevant ones from quasi-static data. These selected feature-base was then separated into a limited number of classes using an ARI-based GK clustering algorithm [16]. Based on the static classifier built in the previous stage, supervised pattern recognition has been used to classify AE data in fatigue tests. It is expected that new AE sources would be developed due to the very own cyclic nature of fatigue test [20]. Therefore, it is supposed to update the classifier by creating a new class for unidentified AE events that possess similar characteristics. Fig. 1 resumes the developed procedure used for the analysis of the AE data, showing its main steps.

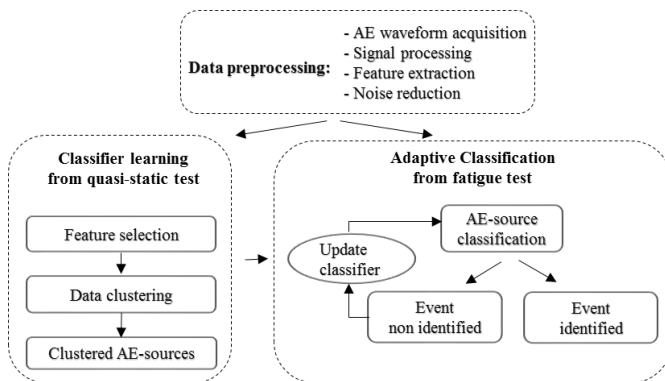


Figure 4. AE data analysis flow chart

III. EXPERIMENTATION

This work deals with the health assessment of tubular composite structures. Such structures are used in many application fields, such as speed rotors, flywheels, pressure vessels, transportation systems and so on. Their stress state is most of the time complex (multiaxial and heterogeneous) due to the combination of loads which it makes particularly difficult the prediction of damage occurrence. In this paper, health was assessed on composite split disks when submitted to quasi-static loading up to failure. The tests were performed according to ASTM D2290 "Apparent hoop tensile strength of plastic or reinforced plastic pipe by split disk method". Rings were produced by cutting and machining filament-wound carbon fibre reinforced epoxy tubular structures intended for the manufacturing of flywheel rotors with a $[(90^\circ)_2 / \pm 45^\circ / (90^\circ)_2]$ lay-up configuration.

The transient elastic waves were recorded during test at the material surface using a multi-channels data acquisition system from EPA (Euro Physical Acoustics) corporation (MISTRAS Group). The system is made up of miniature piezoelectric sensors (micro-80) with a range of resonance of 250 - 325 kHz, preamplifiers with a gain of 40dB and a 20 - 1000 kHz filter, a PCI card with a sampling rate of 1MHz and the AEWin software. The sensors were coupled on the specimen faces using a silicon grease. The calibration of the system was performed after installation of the transducers on the specimen and before each test using a pencil lead break procedure. A part of the ambient noise was filtered using a threshold of 40dB. The acquisition parameters: PDT (Peak Definition Time) = 60 μ sec; HDT (Hit Definition Time) = 120 μ sec and HLT (Hit Lock Time) = 300 μ sec were identified using preliminary measurements. Many features such as absolute energy, counts, hits, amplitude, duration, frequency centroid were calculated from recorded waves.

The detection of damage events, their time sequence, their characterization were determined as far as possible to establish a damage scenario for each specimen using experimental techniques and data, such as infrared thermography, optical observation, analysis of the mechanical behavior of the material during loading and microscopic observations of specimens after rupture or at different loading levels. These experimental techniques are accurately described in a previous paper [21]. Scenarios subsequently established by processing acoustic data will be faced with these reference scenarios.

IV. RESULTS AND DISCUSSION

Noise reduction:

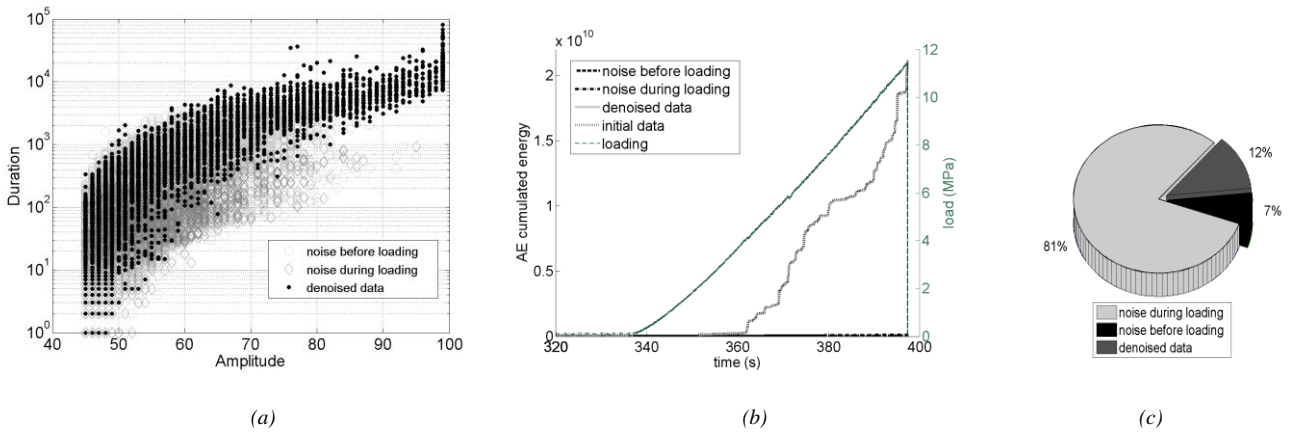


Figure 5. Quasi-static dataset NR 6x90 T1A1: (a) Duration vs. Amplitude; (b) AE cumulated energy; (c) percentage in terms of population

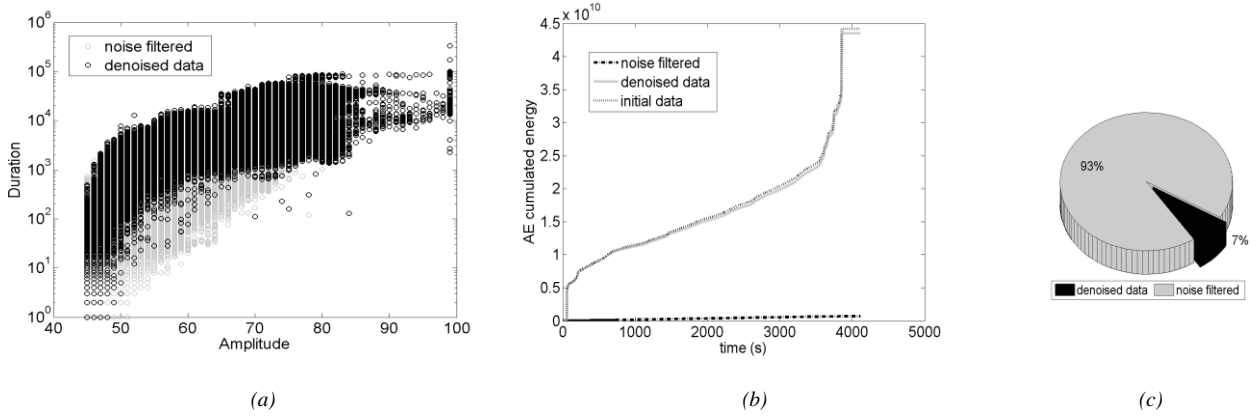


Figure 6. Fatigue dataset NR 6x90 T1A4: (a) Duration vs. Amplitude; (b) AE cumulated energy; (c) percentage in terms of population

Feature selection: Quasi-static dataset NR 6x90 T1A1

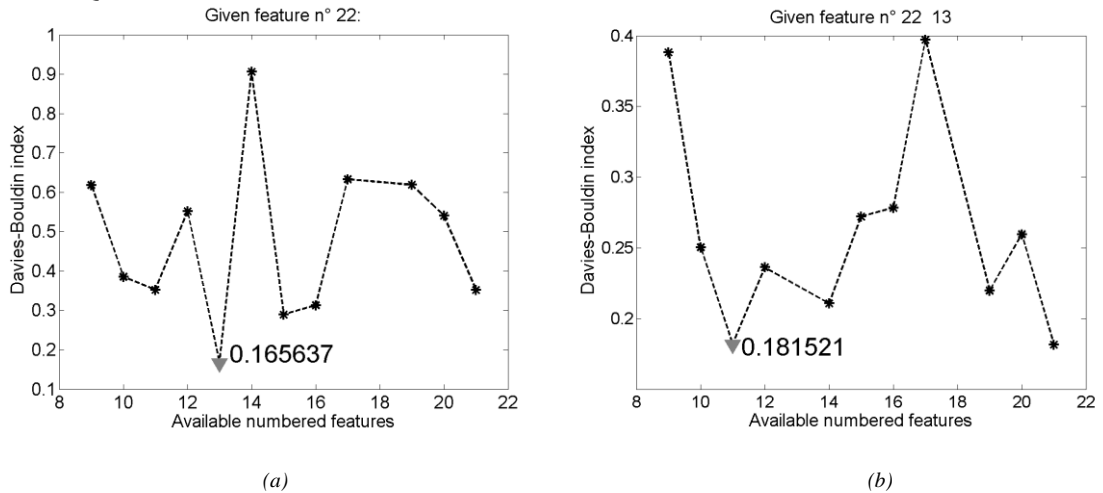


Figure 7. Case of 4 clusters: (a) first selection giving feature n°13 as the best ; (b) second selection giving feature n°11 as the best

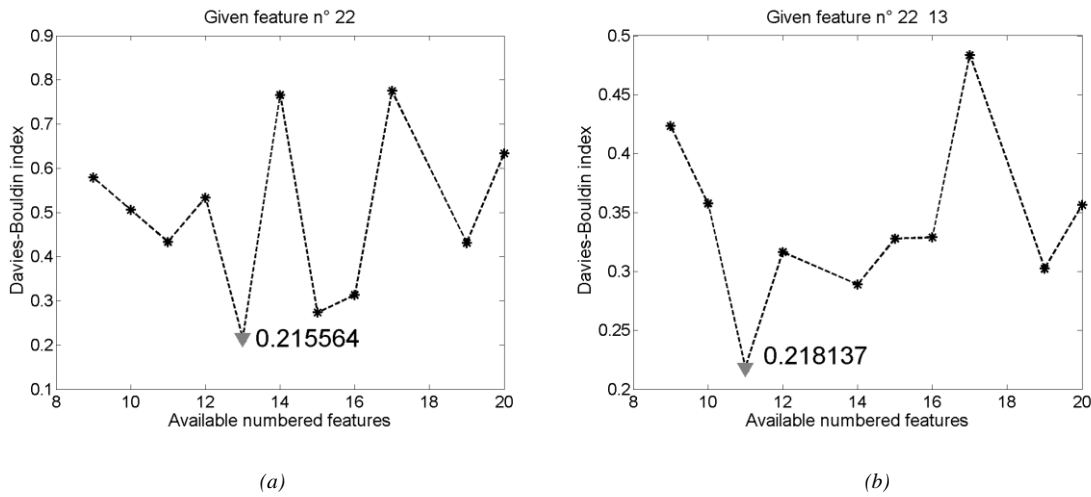


Figure 8. Case of 5 clusters: (a) first selection giving feature n°13 as the best ; (b) second selection giving feature n°11 as the best

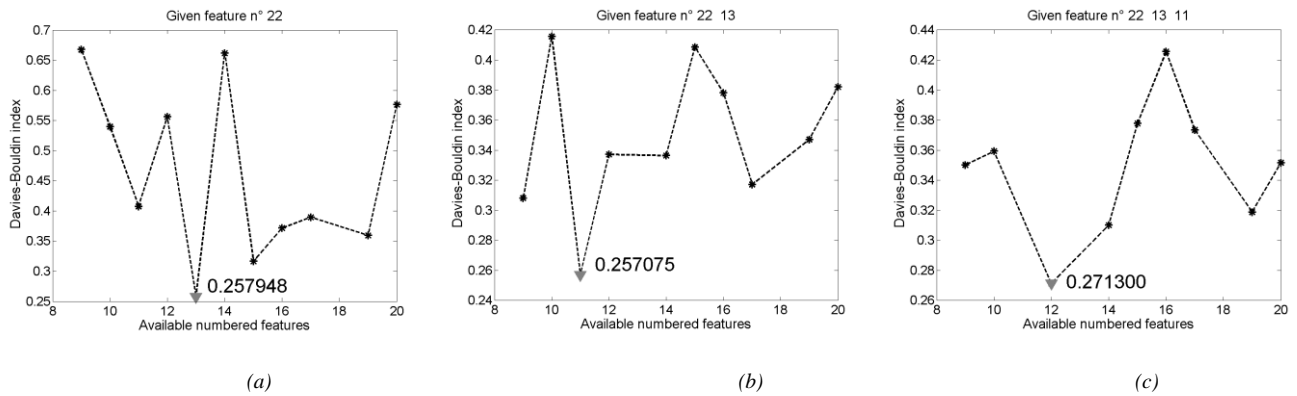


Figure 9. Case of 6 clusters: (a) first selection giving feature n°13 as the best; (b) second selection giving feature n°11 as the best; (c) feature n°12 giving best score in the third selection but no improvement

Remark: three choices of cluster number lead to the same and the best-score subset which is composed of features n° 22, 13 and 11. These ones are so selected for AE data clustering.

Classifier learning from quasi-static dataset NR 6x90 T1A1:

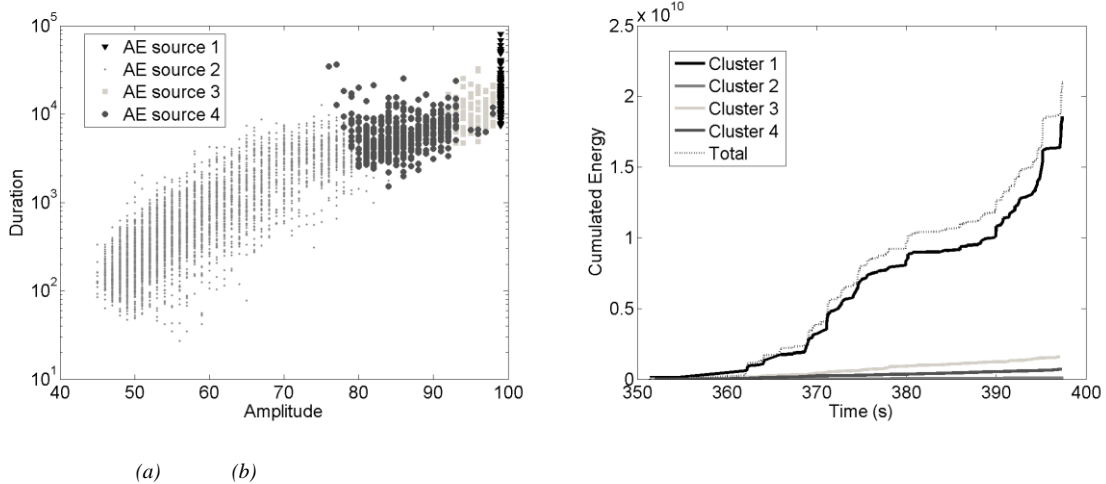


Figure 10. Learning phase: (a) AE data segmentation represented by Duration vs. Amplitude ; (b) Evolution of AE cumulated energy of each AE source

Remark: it is seen that the ARI-based GK algorithm clustering make a good segmentation of AE data in terms of amplitude and AE cumulated energy.

Direct application without updating the classifier to fatigue dataset NR 6x90 T1A4:

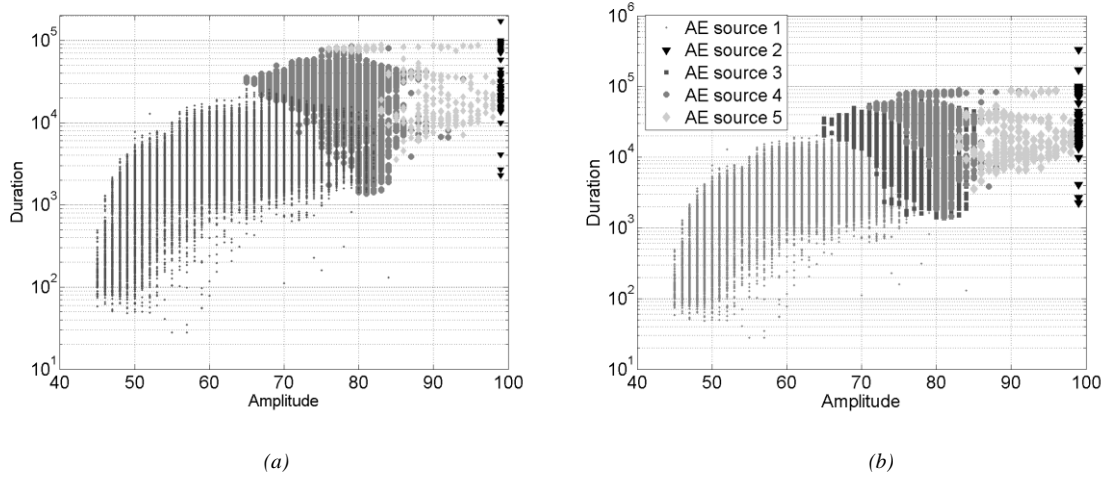


Figure 11. Testing phase: (a) direct classification without adaptation ; (b) adaptive classification giving better separation

Remark: using directly the classifier learnt from quasi-static data to classify AE events originated from fatigue test does not give a good separation of classes (see Fig. 11a) while a better one (see Fig. 11b) is obtained from its updated version with creation of new class give.

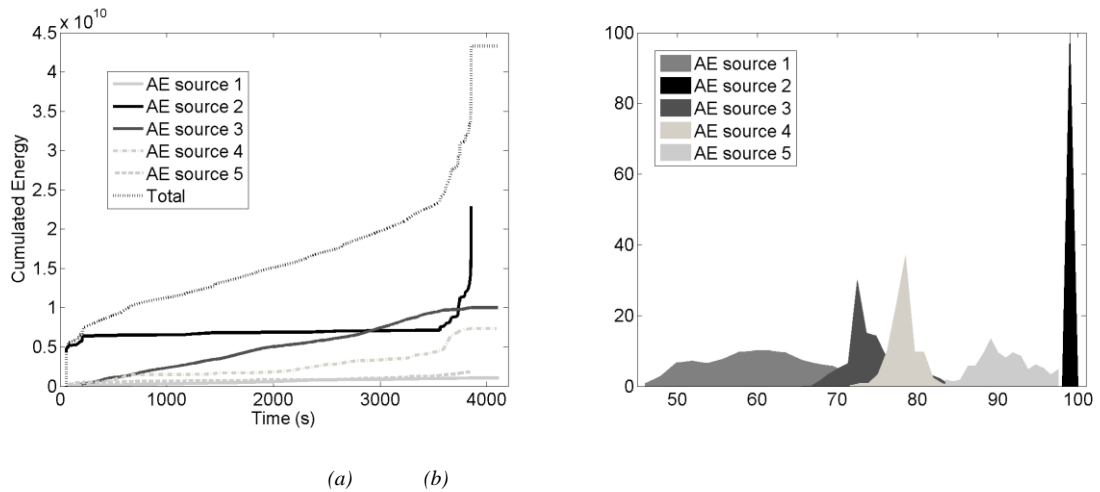


Figure 12. Adaptive classification result: (a) Evolution of AE cumulated energy of each AE source; (b) Amplitude histogram of each AE source

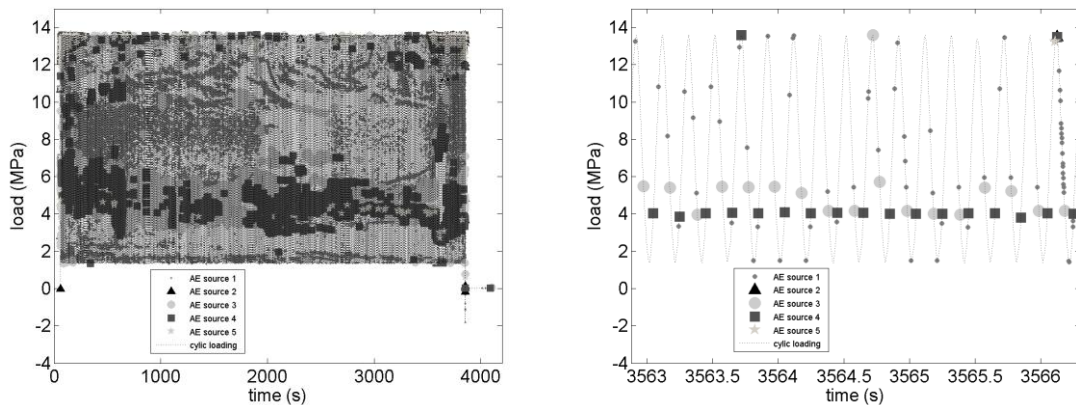


Figure 13. Classified AE events during cyclic loading: (a) global visualization (b) zoomed view of some cycles

V. CONCLUSION

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APPENDIX

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ACKNOWLEDGMENT

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