Vibrations, Shocks and Noise

Robust expansion of mode shapes under epistemic uncertainties

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Highlights

- Expansion methods, robust calibration, uncertainty, info-gap theory, extended constitutive relation error.

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1 Introduction

The present study attempts to leverage an existing non validated numerical model to reconstruct information on unobserved degrees of freedom (dofs) based on the results of modal tests. These methods are referred as expansion methods [1, 2]. An expansion method will be used here and is based on the concept of constitutive relation error (CRE). More precisely, the extended version will be used (ECRE) (see [3, 4] for general descriptions of CRE/ECRE and [5] for more details on how it is used in this work). Since the numerical model is non-validated, the problem we will address is the expansion of mode shapes under epistemic uncertainties (or lack of knowledge). The first objective is to assess the robustness of mode shape expansion in presence of large epistemic uncertainties that are represented as info-gap models. Secondly, a strategy will be presented to maximize the robustness of the expansion by appropriately selecting the model decision variables for a given level of uncertainty. Such an approach is described in section 2. The proposed methodology is finally illustrated on a simple academic test case in section 3.

2 Robust expansion approach

The expansion process depends evidently on the ability of the model to represent the identified structural dynamic behavior. This ability is based not only on the topology of the model but also on the model input parameters which a subset is candidate for model calibration and denoted p. Furthermore, lack of knowledge is commonplace in complex FE model so that the choice of the calibration parameters must take this lack of knowledge into account. Hence, robust expansion process requires not only to minimize the expansion errors but also to enhance the ability of the model to be robust with regards to lack of knowledge in the system [6]. To achieve this goal, info-gap theory is exploited [7]. The uncertain parameters are denoted q and are assumed to be at parameter level. Finally, the expansion process can be expressed as a function of p and u while s denotes the expansion errors:

$$\mathcal{M}(p,q) = s, \quad q \in \mathcal{U}\left(\alpha, q^0\right). \tag{1}$$

The uncertain parameters are modeled by the info-gap relative error model, thought a lot of others models are available (chap. 2 in [7]):

$$\mathcal{U}\left(\alpha, q^0\right) = \left\{ q : \left| \frac{q - q^0}{q^0} \right| \le \alpha \right\}, \tag{2}$$

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where $\alpha \in \mathbb{R}_+$ is the horizon of uncertainty which measures the distance - or the gap - between what is known, the estimate, q^0, and what needs to be known in order to satisfy a given performance measure (equation (3)). More precisely, the model of uncertainty $\mathcal{U}\left(\alpha, q^0\right)$ is an unbounded family of nested convex sets of realizable design. Let s^c be the greatest level of error that the expansion process can tolerate:

$$\mathcal{M}(p,q) = s \leq s^c. \tag{3}$$

The info-gap methodology is a decision theory and one of the main feature in this approach is the robustness function, denoted $\hat{\alpha}$. Broadly speaking, the robustness of decision p is the greatest horizon of uncertainty that can be tolerated without exceeding the critical performance requirement s^c. It is thus written by:

$$\hat{\alpha} = \hat{\alpha}\left(p, s^c\right) = \arg\max_{\alpha \geq 0} \left\{ \max_{q \in \mathcal{U}(\alpha, q^0)} \left\{\mathcal{M}\left(p,q\right)\right\} \leq s^c \right\}. \tag{4}$$

The trade-off between the fidelity-to-data and the robsutness-to-uncertainty (demonstration of the existence of such a trade-off in [6]) can be simply explored by the robustness curve which plots s^c vs. $\hat{\alpha}$. Finally, the robust design is simply the one which maximizes the robustness function:

$$p^R = \arg\max_{p} \left\{\hat{\alpha}\left(p, s^c\right)\right\}. \tag{5}$$

Various examples of robust calibration can be found in [5, 8, 9].

3 Numerical Application

The robust expansion process is illustrated on a simple academic cylinder. The question at stake in this study is: how to calibrate a non-validated model to minimize expansion errors and still be robust with regards to lack of knowledge in the system. The FE model is depicted in Figure 1 while the experimental mode shape are simulated based on the real structure: the experimental mesh is composed of 120 dofs (cf. Figure 2).

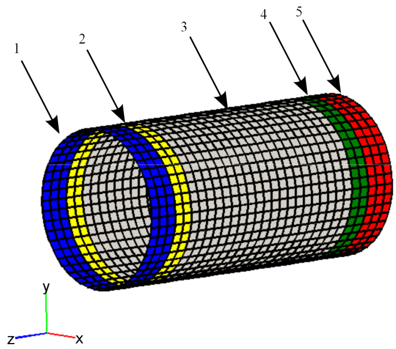

Figure 1: FE model.

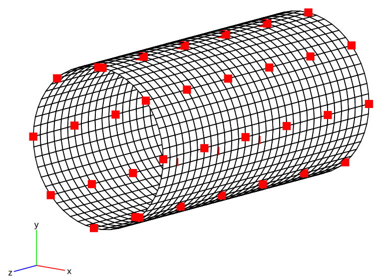

Figure 2: Sensor Locations.

The cylinder is composed of five different zones: they are numbered from 1 to 5 starting from left to right in Figure 1. As expressed in the Table 1, the difference between the real structure, which is used to simulate experimental eigensolutions, and the model one is located in zone 1,2,4 and 5. The zone 3, i.e., the medium zone, is supposed to be completely the same between the real structure and the model.

Name:	Zone	Real	Model	Error
Young Mod. E_1 (Pa):	1	3.10^9	$2{,}5.10^9$	- 6%
Young Mod. E_5 (Pa):	5	3.10^9	$2{,}2.10^9$	- 13%
Young Mod. E_2 (Pa):	2	2.10^9	$2{,}4.10^9$	+ 20%
Young Mod. E_4 (Pa):	4	$2{,}5.10^9$	$2{,}7.10^9$	+ 8%

Table 1: Discrepancy Real/Model.

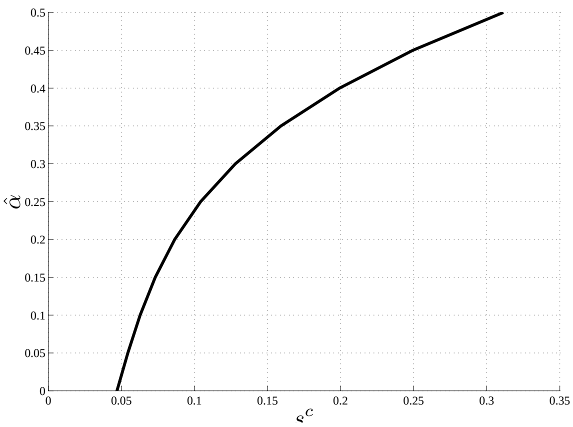

Figure 3: Robustness curve: nominal design.

The parameters to be calibrated are E_2 and E_4 while the lack of knowledge is concentrated in parameters E_1 and E_5. Hence, definitions of q and p are:

$$q = \begin{bmatrix} E_1 & E_5 \end{bmatrix} \qquad p = \begin{bmatrix} E_2 & E_4 \end{bmatrix}. \tag{6}$$

The nominal design is first analyzed in order to assess the robustness of the expansion to lack of knowledge in E_1 and E_5 (cf. Figure 3). Then, an investigation is performed in order to seek the most robust design, that is to say the one which maximizes the robustness function (equation (5) and Figure 4). As expected, some designs are more robust than other ones.

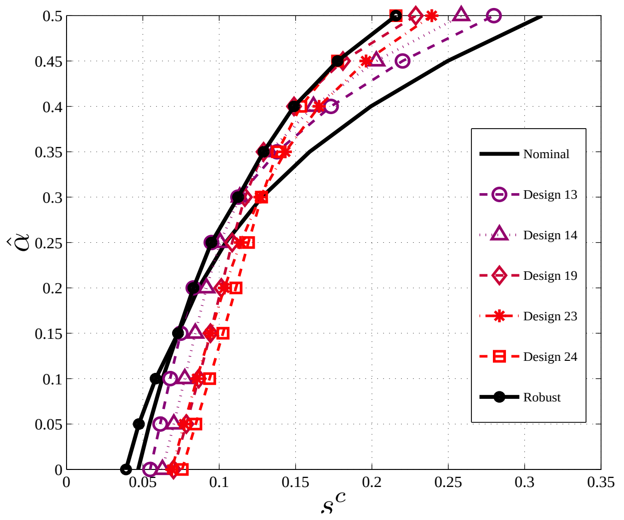

Figure 4: Robustness curves: selected designs.

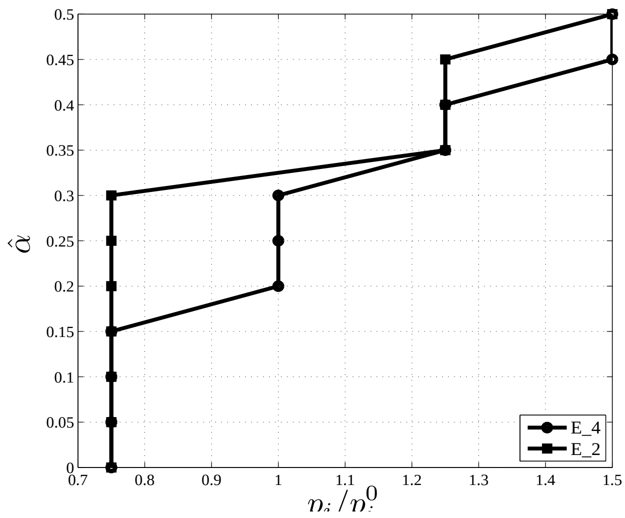

Figure 5: Variations of the most robust designs vs. $\hat{\alpha}$.

A practical application of such curves deals with calibrating a model under uncertainties. For instance, if the experience feedback or the expert judgment pinpoints that the error in the uncertain parameters corresponds with an horizon of uncertainty closed to 0.5, it would be preferable to use the design 24 (rather than the nominal design) in order to minimize the effect of lack of knowledge on the system model predictions (cf. Figure 4). To conclude, the Figure 5 shows the variations of the most robust parameters when the horizon of uncertainty increases. This curve can indicate which calibrated parameters are preferable to use in order to have a model less sensitive to lack of knowledge for each horizon of uncertainty.

4 Conclusions

The objective of this work is to develop and assess a robust mode shape ECRE-based expansion based on a nominal model in presence of large epistemic uncertainty. This proposed approach relies on a robust model calibration to minimize the impact of lack of knowledge in the model on expansion errors.

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