Enhancement of Micro-positioning Accuracy of a Piezoelectric Positioner by Suppressing the Rate-Dependant Hysteresis Nonlinearities

Omar Aljanaideh, Mohammad Al Janaideh, and Micky Rakotondrabe

***Abstract*— Compensation of rate-dependent hysteresis nonlinearities of a Piezoelectric positioner is carried out by integrating the inverse of the rate-dependent Prandtl-Ishlinskii model as a feedforward compensator. The proposed compensator was subsequently implemented to the positioner hardware in the laboratory to study its potential for rate-dependent hysteresis compensation on a real-time basis. The experimental results obtained under different excitation frequencies revealed that the integrated inverse compensator can substantially suppress the hysteresis nonlinearities in the entire frequency range considered in the study. The proposed inverse model compensates for the rate-dependent hysteresis nonlinearities without using the feedback control techniques.**

I. Introduction

Piezoelectric actuators are becoming increasingly popular for use in micro- and nano-positioning applications because of a number of advantages which include nanometer resolution, high stiffness, and fast response. These smart actuators have been employed in different applications such as micromanipulation [1], observing and manipulating objects at the nanoscale [2][3], design and control of smart micro-and nano-positioning systems, and vibration control [4]. However, piezo micro-positioning actuators exhibit hysteresis nonlinearities between the applied input voltage and the output displacement [4]. These nonlinearities have been associated with oscillations in the open-loop system's responses, and poor tracking performance and potential instabilities in the closed-loop system. Piezo micro-positioning actuators, similar to other smart actuators exhibit an increase in the hysteresis nonlinearities when the excitation frequency of the applied input voltage increases, see for example [5][6].

A number of rate-independent hysteresis models have been proposed to characterize hysteresis nonlinearities of piezo micro-positioning actuators. These models include: the Preisach model [2][7], the Prandtl-Ishlinskii model [8] and the Bouc-Wen model [9]. Such models are very efficient when the actuators/positioners work at low frequency where the hysteresis is rate-independent. However, in practical situations, when higher excitation frequencies are applied, rate-dependent hysteresis nonlinearities are observed. Modeling and compensation of rate-dependent hysteresis can be carried out using two approaches. The first approach incorporates approximating the rate-dependent hysteresis as a cascade arrangement of a rate-independent hysteresis model and a linear dynamics, which is the so-called Hammerstein approximation. However, this approach requires the employment of two different feedforward compensators: one compensator for the hysteresis and another compensator for the dynamics [4]. The second approach incorporates the employment of feedback control schemes. In this technique, a rate-independent hysteresis compensator is first applied in order to observe input-output linearity. Then, a linear feedback scheme is used to improve the dynamics [10]. This necessitates installation of feedback sensors, which is a challenging task for the above mentioned applications due to space limitations and small sizes (micromanipulation, microassembly ...). The available feedback sensors are not convenient for such purposes. Embeddable sensors (strain gage), for example, cannot furnish the required performances, while the precise and high bandwidth sensors are expensive and bulky (optical sensors, ...).

In this study, we propose a novel and crucial compensation algorithm to compensate for rate-dependent hysteresis in a piezoelectric positioner at different excitation frequencies. Rate-dependent hysteresis nonlinearities are characterized using the rate-dependent Prandtl-Ishlinskii model. The analytical inverse of the rate-dependent Prandtl-Ishlinskii model is formulated and applied as a feedforward compensator to compensate for the rate-dependent hysteresis nonlinearities in the considered positioner. The main advantage of the rate-dependent Prandtl-Ishlinskii model over the other hysteresis models used in the literature is that its inverse can be obtained analytically, which can be employed as feedforward compensator to control the piezoelectric positioner over different excitation frequencies without feedback control techniques.

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In [11], the analytical inverse of the Prandtl-Ishlinskii model constructed with time-dependent thresholds has been proposed. The explicit inversion formula for the Prandtl-Ishlinskii model remains applicable, provided that the distances between the time-dependent thresholds do not decrease in time. In this paper we use this inverse as a feedforward controller in order to compensate for the rate-dependent hysteresis nonlinearities of a piezoelectric positioner.

The contents of the paper are as follows. The piezoelectric positioner system is presented in Section II. Then, we characterize its rate-dependent hysteresis nonlinearities in Section III. In Section IV, the modeling and the compensation of the rate-dependent hysteresis nonlinearities are carried out at different excitation frequencies. Both theoretical and experimental aspects are discussed in the same section. Finally, Section V concludes the paper.

II. Presentation of the experimental setup

The piezoeletric positioner used for the experiments in this paper has a cantilevered structure. Such positioners are widely used as precise manipulators in micromanipulation and microassembly of small objects [1], as actuators in miniaturized bio-inspired robots [12], as scanners in atomic force microscopy (AFM) [13], or also as actuators in micromirror orientation [14]. The high resolution of positioning (in the order of nanometer), the high bandwidth and the high stiffness make them very recognized for these applications. However, as we will see in the next section, these positioners are typified by hysteresis nonlinearities that increases with the excitation frequency of the applied input.

The experimental study was performed on a positioner with several layers (multilayer) in order to obtain a high range of bending of the cantilever with low voltages. The layers are based on lead zirconate titanate (PZT) ceramics and are glued themselves. When the same input voltage $u(t)$ is applied to all the layers, a bending $y(t)$ of the cantilever is obtained (Fig. 1-a). The experimental setup is presented in Fig. 1-b and is composed of:

- a multilayered piezoeletric cantilevered positioner (with 36 layers) having total and active dimensions (active length × width × total thickness) of: $15mm \times 2mm \times 2mm$,
- an optical displacement sensor (Keyence, model LC2420) was used to measure the bending of the cantilever. The sensor is tuned to have a resolution of $10nm$, a precision of $100nm$ and a bandwidth more than $1kHz$,
- a computer with Matlab-Simulink software used to manage the different signals (input voltage, reference input, output displacement) and to implement the future feedforward controller,
- and a dSPACE board that serves as DAC/ADC converter between the computer and the rest of the experimental setup. The refresh time of the computer and dSPACE board is set equal to $0.2ms$ which permits to account the bandwidth of the positioner.

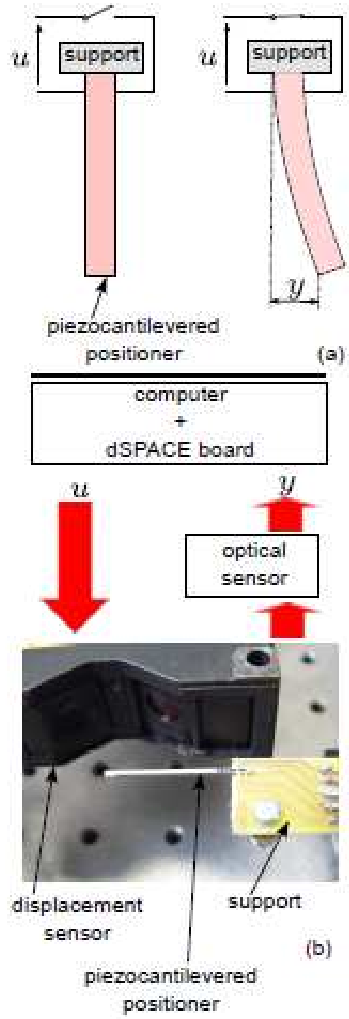

Fig. 1: a: A piezoelectric cantilevered positioner. b: presentation of the setup.

Fig. 2 shows the output displacement of the piezoelectric positioner obtained when an input voltage of $u(t) = 10sin(2\pi ft)$ was applied at excitation frequency $f = 10$, 50, 100, and 200 Hz.

III. Hysteresis Modeling

The rate-dependent Prandtl-Ishlinskii model is used to characterize the rate-dependent hysteresis nonlinearities. This model is formulated in [11] using the concept of play operator with rate-dependent threshold. The model and its inverse are applied to characterize and compensate for the rate-dependent hysteresis nonlinearities.

A. The model

In this paper, we deal with the space $AC(0,T)$ of real absolutely continuous functions defined on the interval

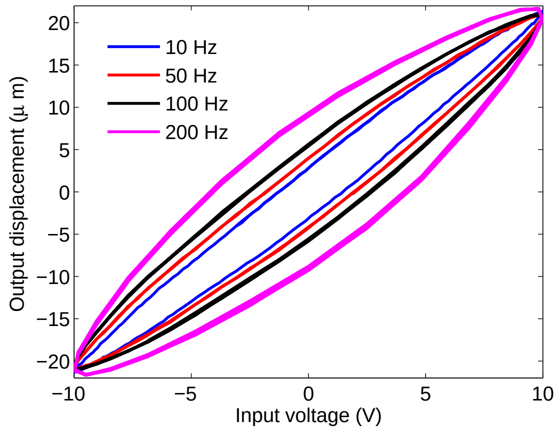

Fig. 2: The relationship between the input voltage u and the output displacement y of the piezoeletric positioner at different excitation frequencies (experimental characterization results).

$[0,T]$. For an input $u(t) \in AC(0,T)$, the output of the rate-dependent Prandtl-Ishlinskii model is constructed based on the rate of the applied input $\dot{u}(t)$, which can be expressed as

$$\Psi[u](t) = a_0 u(t) + \sum_{i=1}^{n} a_i \Phi_{r_i(\dot{u}(t))}[u, x_i](t) \tag{1}$$

where n represents the number of the time-dependent play operators considered in the model. We denote the output of the time-dependent play operator as

$$z_i(t) = \Phi_{r_i(\dot{u}(t))}[u, x_i](t), \tag{2}$$

where x_i are given initial conditions for $i = 1, 2, \ldots, n$ such that for $i = 1, \ldots, n-1$ we have

$$\begin{aligned} &|x_1| \leq r_1(\dot{u}(0)), \\ &|x_{i+1} - x_i| \leq r_{i+1}(\dot{u}(0)) - r_i(\dot{u}(0)). \end{aligned} \tag{3}$$

The dynamic thresholds $r_i(t)$ are defined for $t \in [0,T]$ as

$$0 \leq r_1(\dot{u}(t)) \leq r_2(\dot{u}(t)) \leq \cdots \leq r_n(\dot{u}(t)). \tag{4}$$

The exact inversion formula for the rate-dependent model holds under the condition that the distances between the dynamic thresholds $r_i(\dot{u}(t))$ do not decrease in time [11]. Analytically for $\forall i = 1, \ldots, n-1$

$$\frac{d}{dt}(r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t))) \geq 0. \tag{5}$$

We propose the following dynamic thresholds

$$r_i(\dot{u}(t)) = \alpha_i + \kappa(\dot{u}(t)). \tag{6}$$

This can be shown to be mathematically equivalent to modeling hysteresis and creep by means of an analogical model with elastic, plastic, and viscous elements [15]. We also have,

$$\alpha_i - \alpha_{i-1} \geq \sigma, \tag{7}$$

where σ is a positive constant. The constants α_i in Eq. (6) represent the rate-independent hysteresis effects, while the function $\kappa(\dot{u}(t))$ is proposed to characterize the rate-dependent hysteresis effects. With this choice

$$r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t)) = \alpha_i - \alpha_{i-1} \tag{8}$$

and

$$\frac{d}{dt}(r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t))) = 0. \tag{9}$$

From these equations it can be concluded that the exact inversion formula for the time-dependent Prandtl-Ishlinskii model holds. The dynamic threshold of the rate-dependent play operator can be presented for $i = 1, 2, \ldots, n$

$$\alpha_i = \zeta i \tag{10}$$

where ζ is a positive constant. The function $\kappa(\dot{u}(t))$ can be chosen as

$$\kappa(\dot{u}(t)) = \beta |\dot{u}(t)| \tag{11}$$

where β is a positive constant and $r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t)) = \zeta$.

In the time dependent play operator, an increase in input $u(t)$ causes the output of the play operator $z(t)$ to increase along the curve $u(t) - r_i(\dot{u}(t))$, while a decrease in input $u(t)$ causes the output to decrease along the curve $u(t) + r_i(\dot{u}(t))$, resulting in a symmetric hysteresis loop. As shown in [5], the rate-dependent Prandtl-Ishlinskii model can characterize rate-dependent hysteresis nonlinearities in piezo micro-positioning actuators over a range of different excitation frequencies. In this paper, we show that the inverse of the rate-dependent Prandtl-Ishlinskii model is achievable and can be applied as a feedforward compensator to compensate for rate-dependent hysteresis nonlinearities in real-time systems.

B. Parameter Identification

The measured rate-dependent hysteresis loops of the piezo micro-positioning actuator presented in Fig. 2 are used to identify the parameters of the rate-dependent Prandtl-Ishlinskii model and its inverse. We choose the dynamic threshold of Eq. (6) and Eq. (11) to model the rate-dependent hysteresis in the piezo micro-positioning actuator. The characterization error of the rate-dependent Prandtl-Ishlinskii model is defined as

$$E(k) = \Psi[u](k) - y(k), \tag{12}$$

where $y(k)$ represents the measured displacement of the piezo micro-positioning actuator when an input voltage at a particular excitation frequency is applied and $\Psi[u](k)$ is the output of the rate-dependent Prandtl-Ishlinskii model under the same input voltage. The index k ($k = 1, \ldots, K$) refers to the number of data points considered in computing the error for one complete hysteresis loop. The parameter vector $X = \{\beta, \zeta, a_0, a_1, a_2, \ldots, a_n\}$ of the rate-dependent Prandtl-Ishlinskii model Ψ, was identified through minimization of characterization error function over different excitation frequencies, given by

$$Q(X) = \Theta(k). \tag{13}$$

The model error function Θ is used to identify the parameters of the rate-dependent Prandtl-Ishlinskii model Ψ. The error function Θ is expressed as

$$\Theta(k) = \sum_{k=1}^{K} (\Psi[u](k) - y(k))^2. \tag{14}$$

The model error function is constructed through summation of squared errors over a range of input frequencies. 100 data points ($K = 100$) were available for each measured hysteresis loop. Owing to the higher error at higher excitation frequencies, a weighting constant A_p was introduced to emphasize the error minimization at higher excitation frequencies. The error minimization is performed using the MATLAB constrained optimization toolbox and considering the following constraints

$$\{\beta, \quad \zeta\} > 0,$$

$$\sum_{i=0}^{j} a_i > 0 \quad \text{for} \quad j = 0, \ldots, n$$

The results suggest that a rate-dependent Prandtl-Ishlinskii model formulated using ten rate-dependent play hysteresis operators ($n = 10$) yields reasonable agreement with the experimental measurements. The parameters of the model are:$\zeta = 0.42067$, $\beta = 4.1317 \times 10^{-4}$, $a_0 = 0.8515$, $a_1 = 0.6893$, $a_2 = 0.01442$, $a_3 = 0.2657$, $a_4 = 0.0111$, $a_5 = 0.1761$, $a_6 = 0.0001$, $a_7 = -0.0573$, $a_8 = 0.0007$, $a_9 = -0.5000$, and $a_{10} = 0.8895$. The rate-dependent Prandtl-Ishlinskii model is used to characterize the rate-dependent hysteresis nonlinearities of the piezoelectric positioner. Fig. 3 shows the hysteresis with different frequencies obtained by simulation of the identified rate-dependent model. By comparing Fig. 3 with the experimental characterization in Fig. 2, we can see that the identified model fits well with the experiments.

IV. Compensation of Rate-Dependent Hysteresis Nonlinearities

The concept of the open-loop control system used in this paper is to obtain an identity mapping between the input reference $y_r(t)$ and the output $y(t)$ such that $y_r(t) = y(t)$ at different frequencies (rate-dependent hysteresis compensation) as pictured in Fig. 4. When the output of the exact inverse of the rate-dependent Prandtl-Ishlinskii model $\Psi^{-1}[y_r](t)$ is applied as a feedforward controller to compensate for the rate-dependent hysteresis nonlinearities presented by the rate-dependent Prandtl-Ishlinskii model $\Psi[u](t)$, the output of the compensation is expressed as

$$y(t) = \Psi \circ \Psi^{-1}[y_r](t). \tag{15}$$

The output of the inverse is expressed as

$$\Psi^{-1}[y_r](t) = b_0 y_r(t) + \sum_{i=1}^{n} b_i \Phi_{s_i(\dot{y}_r(t))}[y_r, z_i](t). \tag{16}$$

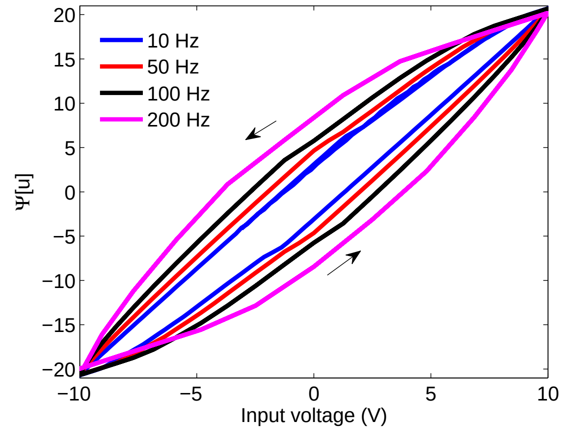

Fig. 3: The relationship between the input voltage u and the output Ψ of the rate-dependent Prandtl-Ishlinskii model at different excitation frequencies (simulation of the identified rate-dependent model).

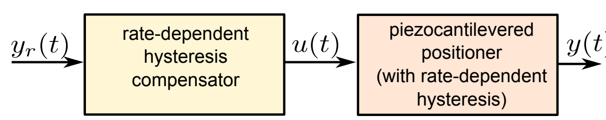

Fig. 4: Block scheme of the compensation of the rate-dependent hysteresis.

where z_i are initial conditions that will be defined later.

We denote the output of the rate-dependent play operator of the inverse model by

$$u_i(t) = \Phi_{s_i(\dot{y}_r(t))}[y_r, z_i](t). \tag{17}$$

The thresholds of the inverse model are

$$s_1(\dot{y}_r(t)) = a_0 r_1(\dot{y}_r(t)), \tag{18}$$

$$s_{i+1}(\dot{y}_r(t)) - s_i(\dot{y}_r(t)) = \left(\sum_{j=0}^{i} a_j\right)(r_{i+1}(\dot{y}_r(t)) - r_i(\dot{y}_r(t)) \tag{19}$$

The weights of the inverse model $b_0, b_1, \ldots, b_n$ are

$$b_0 = \frac{1}{a_0}, \tag{20}$$

$$b_i = \frac{1}{\sum_{j=0}^{i} a_j} - \frac{1}{\sum_{j=0}^{i-1} a_j}. \tag{21}$$

The initial conditions of the inverse model $z_1, z_2, \ldots, z_n$ are

$$z_1 = a_1 x_1, \tag{22}$$

$$z_{i+1} - z_i = \left(\sum_{j=0}^{i} a_j\right)(x_{i+1} - x_i). \tag{23}$$

The parameters of the inverse model obtained by Eq. (21) are $b_0 = 1.1742$, $b_1 = -0.3457$, $b_2 = -0.004$, $b_3 = -0.0730$, $b_4 = -0.0026$, $b_5 = -0.0410$, $b_6 = -0.0001$, $b_7 = 0.0122$, $b_8 = -0.0001$, $b_9 = 0.1094$, $b_{10} = -0.2616$.

A. Compensation results

The calculated rate-dependent hysteresis compensator has been implemented in the computer - dSPACE and applied to the piezoelectric positioner following the block scheme in Fig. 4. Fig. 5 shows the relationship between the desired displacement y_r and the output y of the piezoelectric positioner when the inverse rate-dependent Prandtl-Ishlinskii model is applied at different excitation frequencies. Fig. 6 shows the time history of the desired input displacement and output displacement at different excitation frequencies. Fig. 7 shows the time history of the positioning error. Since the initial hysteresis was up to $\frac{17.5}{40} = 43.75\%$ (see Fig. 2) in the absence of the compensator, and reduced to lower than $\frac{2}{40} = 5\%$ with the proposed compensator for a frequency less than $100Hz$ and to lower than $\frac{4}{40} = 10\%$ for a frequency over $100Hz$ when using the proposed compensator, we can deduce the that the proposed inverse model can effectively compensate for the rate-dependent hysteresis of the actuator.

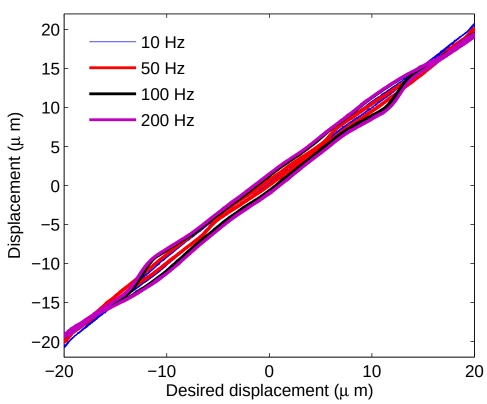

Fig. 5: The relationship between the desired displacement y_r and the output y of the piezoelectric positioner at different excitation frequencies.

At low excitation frequencies, the rate-dependent Prandtl-Ishlinskii model and its inverse, constructed based on the dynamic threshold Eq. (10), are reduced to the rate-independent Prandtl-Ishlinskii model and its inverse. Analytically, the dynamic thresholds are reduced to

$$r(\dot{u}(t)) \approx \zeta i. \tag{24}$$

Consequently, the suggested model can be considered as an extension for the rate-independent version of the model. Fig. 8 shows the compensation results at low excitation frequencies where the hysteresis is rate-independent. This demonstrates the high accuracy obtained at low frequency.

V. Conclusion

This paper presented the modeling and the feedforward control of hysteresis nonlinearities of a piezoelectric positioner with cantilever structure. The piezoelectric positioner shows strong rate-dependent hysteresis at high excitation frequencies which substantially reduce the actuator accuracy. The rate-dependent Prandtl-Ishlinskii model can accurately describe the rate-dependent hysteresis of the piezoelectric positioner. The experimental results demonstrate that the inverse of purposed model can effectively compensate for the rate-dependent hysteresis of the piezoelectric positioner.

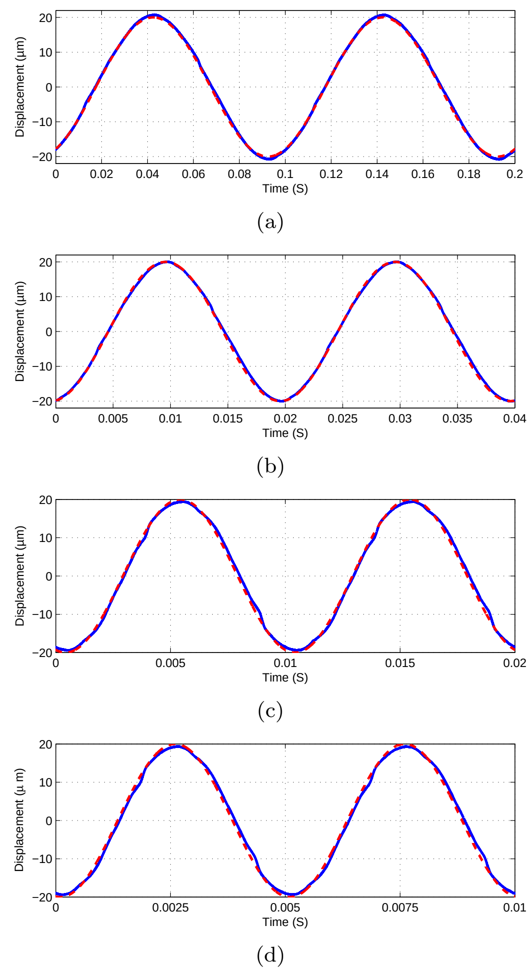

Fig. 6: The time history of the desired input displacement (dashed line) and output displacement (solid line) at (a) 10 Hz, (b) 50 Hz, (c) 100 Hz, and (d) 200 Hz.

Acknowledgment

This work is partially supported by the national ANR-JCJC C-MUMS-project (National young investigator project ANR-12-JS03007.01: Control of Multivariable Piezoelectric Microsystems with Minimization of Sensors).

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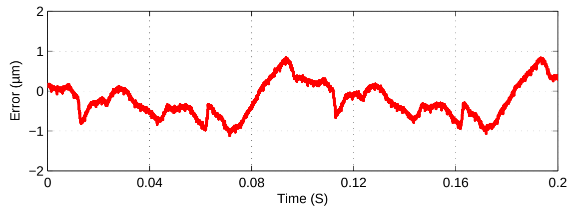
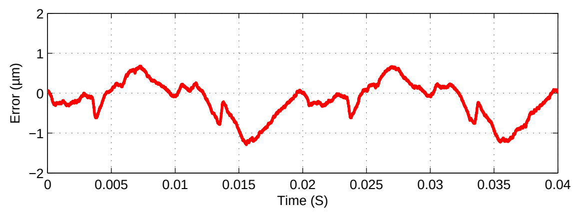


(a)



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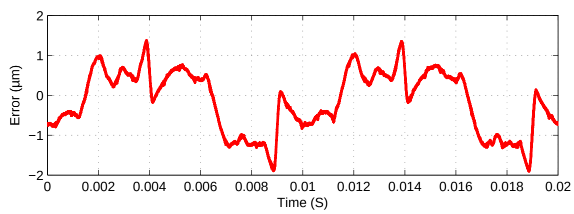

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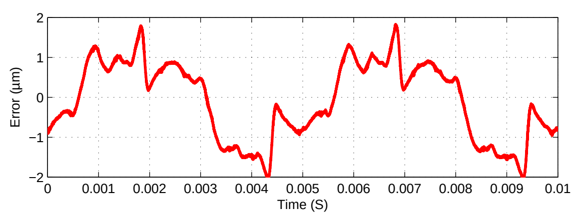
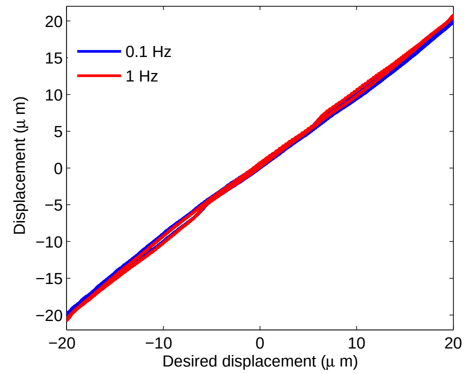
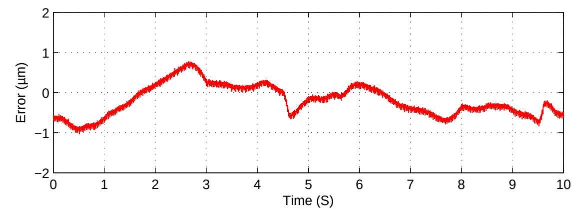


(d)

Fig. 7: The time history of the positioning error at (a) 10 Hz, (b) 50 Hz, (c) 100 Hz, and (d) 200 Hz.



(a)



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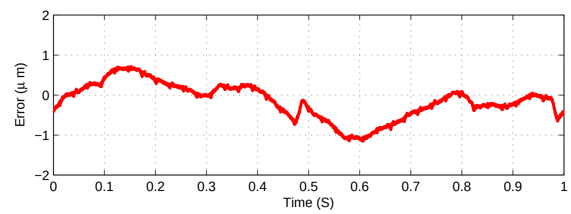

(c)

Fig. 8: (a) The relationship between the desired displacement and the output of the piezoelectric positioner at 0.1 and 1 Hz excitation frequencies, (b) shows the psoitioning error at 0.1 Hz, (c) shows the positioning error at 1 Hz.

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