Model updating of locally nonlinear systems based on Multi-harmonic Extended Constitutive Relation Error

I. Isasa[1], **A. Hot**[2], **S. Cogan**[2], **E. Sadoulet-Reboul**[2]

[1] Orona, Department of Mechanical Engineering,
Pol. Lastaola, 20120, Hernani, Spain

[2] FEMTO-ST Institute, Structural Dynamics Research Group,
Rue de l'Epitaphe 24, 25000, Besançon, France

e-mail: **aurelien.hot@univ-fcomte.fr**

Abstract

Improving the fidelity of numerical simulations using available test data is an important activity in the overall process of model verification and validation. While model updating or calibration of linear elastodynamic behaviors has been extensively studied for both academic and industrial applications over the past three decades, methodologies capable of treating nonlinear dynamics remain relatively immature. The authors propose a novel strategy for updating an important subclass of nonlinear models characterized by globally linear stiffness and damping behaviors in the presence of local nonlinear effects. Existing nonlinear updating strategies are based on the Response Force Surface (RSF), Proper Orthogonal Decomposition (POD), or first-order Harmonic Balance (HB) methods. With the exception of the RFS approach, these methods introduce some form of linearization and this naturally limits their application to relatively weak nonlinear effects. As for the RFS approach, its major weakness lies in the fact that it requires that the structural responses be measured on all model degrees-of-freedom where significant nonlinear effects are present. In this paper, a novel methodology is presented which effectively combines two well-known methods for structural dynamic analysis: the Multi-harmonic Balance method for calculating the periodic response of a nonlinear system and the Extended Constitutive Relation Error method for establishing a well-behaved metric for modeling and test-analysis errors. The proposed methodology neither requires any linear approximation nor the observation of all nonlinear degrees-of-freedom. The advantages and limitations of the proposed nonlinear updating strategy will be illustrated based on an academic example.

1 Introduction

Nonlinear phenomena are commonplace in mechanical systems containing mechanisms, joints and contact interfaces [1]. Engineers often simplify the behavior of complex structural models by considering them to be linear for dynamic analyses, thus neglecting nonlinear effects due to large displacements, contact, clearance and impact phenomena, among others.

The following paper is devoted to the revision of nonlinear models in the field of structural dynamics based on measured responses. During the past two decades, linear model updating has been extensively studied to improve the accuracy of simulations [2]. Nonlinear model updating techniques on the other hand have received much less attention. Both time domain or frequency domain approaches can be found in the literature. In the time domain, the Restoring Force Surface method (RFS) and Proper Orthogonal Decomposition (POD) are described in detail in the overview paper by Kerschen et al. [3] with complete references to the literature. More recently, Gondhalekar et al. has proposed a strategy combining the RFS method with model reduction [4]. In the frequency domain, Böswald and Link [5] have developed a methodology based on the first order Harmonic Balanced method to get a suitable representation of nonlinear effects and they have applied their approach to update nonlinear joint parameters in complex structural assemblies. Another frequency domain method is investigated by Puel [6] where the Extended Constitutive Relation Error (ECRE) for linear dissipative systems is generalized to nonlinear model updating with a first order harmonic balance approximation. With the exception of the RFS method, the existing methods for nonlinear updating are based on some form of linearization and this naturally limits their application to relatively weak nonlinear effects. As for the RFS approach, its major weakness lies in the fact that it requires that the structural responses be measured on all model degrees-of-freedom where significant nonlinear effects are present.

In this paper, a novel methodology is presented which effectively combines the Multi-harmonic Balance method for calculating the periodic response of a nonlinear system and the Extended Constitutive Relation Error method for establishing a well-behaved metric for modeling and test-analysis errors. The proposed approach is not based on any linear approximations and does not require the observation of all nonlinear degrees-of-freedom.

2 Mathematical formulation

2.1 Equations of motion

The equations of motion of a discrete linear structure can be written:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = p(t) \tag{1}$$

where, $K, M, C \in \Re^{N,N}$ are respectively the symmetric stiffness, mass and damping matrices, with the stiffness matrix assumed to be non-negative definite; $p(t) \in \Re^{N,1}$ is a vector of external forces; $q(t) \in \Re^{N,1}$ is the vector of time responses on the N degrees-of-freedom (dofs).

The equation of motion of a nonlinear structure can be written in the same way as a linear structure with the addition of a nonlinear term, $f_{NL}(q(t), \dot{q}(t)) \in \Re^{N,1}$, which can depend on the system displacements and velocities:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) + f_{NL}(q(t), \dot{q}(t)) = p(t) \tag{2}$$

The origin of these nonlinear forces can be quite diverse, including:

- Some large displacement systems, for example the classical pendulum.
- Material nonlinearities including locally plastic or viscoplastic behaviors, shape memory alloys, and so on.
- Local interface nonlinearities including Hertz contact, dry friction, intermittent contact or clearance phenomena.

The response of a nonlinear system can be qualitatively very different from a linear one. In a linear system the steady-state response to a periodic excitation is at the same frequency as the excitation force once the transient term vanishes in time and is independent of the initial conditions. The periodic response of a nonlinear system, when it exists, generally exhibits primary and secondary resonances and may depend on the initial conditions [7].

Although transient behavior may be important, the study of periodic solutions and their stability remains essential to capture the behavior of a vibrating system. Nonlinear time domain simulations are extremely burdensome especially when they are used to calculate the steady state response of large-order models. The Multi-harmonic Balance method is based on a Fourier series approximation and was developed with the objective of solving the periodic response of nonlinear systems more efficiently.

2.2 Multi-harmonic balance method

The MHB method is a frequency domain approach developed to solve equation (2) for a periodic excitation. Many extensions to the first-order harmonic balance approach to include higher harmonics were developed in the 1980's, for example [8] or [9]. We have based our developments on the formulation proposed by Cardona et al. [10] and more recently applied to complex industrial structures with contact effects by Petrov et al., [11].

The equilibrium equation of a nonlinear system of N degrees-of-freedom is given by equation (2). Expressing the vector of time responses $q(t)$ as a Fourier series yields:

$$q(t) = Q_0 + \sum_{j=1}^{n} \left(Q_j^c \cos m_j \omega t + Q_j^s \sin m_j \omega t \right) \tag{3}$$

where:

- Q_0 represents the constant or static contribution
- Q_j^c and Q_j^s are respectively the j^{th} cosine and sine coefficients of the Fourier series
- m_j expresses the harmonic of the excitation frequency ω

Introducing this expression into equation (2) yields:

$$\begin{gathered} K \left(Q_0 + \sum_{j=1}^{n} Q_j^c \cos m_j \omega t + Q_j^s \sin m_j \omega t \right) + \\ C \left(\sum_{j=1}^{n} -m_j \omega \; Q_j^c \sin m_j \omega t + m_j \omega \; Q_j^s \cos m_j \omega t \right) + \\ M \left(\sum_{j=1}^{n} -(m_j \omega)^2 \; Q_j^c \cos m_j \omega t - (m_j \omega)^2 \; Q_j^s \sin m_j \omega t \right) + \\ f(q(t), \dot{q}(t)) - p(t) = 0 \end{gathered} \tag{4}$$

A Galerkin procedure is then applied by sequentially pre-multiplying equation (4) by the harmonic functions $(1, \cos \omega t, \sin \omega t, \cos m_1 \omega t, \ldots, \cos m_n \omega t, \sin m_n \omega t)$ and integrating over the period $T = 2\pi/\omega$.

Regrouping the resulting equations for each harmonic in the Fourier expansion, the following frequency domain expression can be obtained:

$$\mathcal{Z}(\omega)\mathcal{Q} + \mathcal{F}(\mathcal{Q}) - \mathcal{P} = 0 \tag{5}$$

where $\mathcal{Q} = \{Q_0 \; ; \; Q_1 \; ; \; Q_2 \; ; \; \cdots \; ; \; Q_{2n-1} \; ; \; Q_{2n}\}$ is the vector of harmonic coefficients with $Q_i \in \Re^{N,1}$. The matrix $\mathcal{Z} \in \Re^{(2n+1)N,(2n+1)N}$ is given by:

$$\mathcal{Z} = \begin{bmatrix} K & 0 & 0 & \cdots & 0 & 0 \\ 0 & K-(m_1\omega)^2 M & m_1 \omega C & \cdots & 0 & 0 \\ 0 & -m_1 \omega C & K-(m_1\omega)^2 M & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & K-(m_n\omega)^2 M & m_n \omega C \\ 0 & 0 & 0 & \cdots & -m_n \omega C & K-(m_n\omega)^2 M \end{bmatrix} \tag{6}$$

and the vectors $\mathcal{F};\mathcal{P} \in \Re^{(2n+1)N,1}$ corresponding respectively to the nonlinear forces and the external excitations are given by:

$$\mathcal{F} = \left\{ \begin{array}{c} \int_0^T f_{NL}(q(t),\dot{q}(t))\ dt \\ \frac{\omega}{\pi} \int_0^T f_{NL}(q(t),\dot{q}(t)) \cos\omega t\ dt \\ \frac{\omega}{\pi} \int_0^T f_{NL}(q(t),\dot{q}(t)) \sin\omega t\ dt \\ \vdots \\ \frac{\omega}{\pi} \int_0^T f_{NL}(q(t),\dot{q}(t)) \cos m_n\omega t\ dt \\ \frac{\omega}{\pi} \int_0^T f_{NL}(q(t),\dot{q}(t)) \sin m_n\omega t\ dt \end{array} \right\} \tag{7}$$

and

$$\mathcal{P} = \left\{ \begin{array}{c} \int_0^T p(t)\ dt \\ \frac{\omega}{\pi} \int_0^T p(t) \cos\omega t\ dt \\ \frac{\omega}{\pi} \int_0^T p(t) \sin\omega t\ dt \\ \vdots \\ \frac{\omega}{\pi} \int_0^T p(t) \cos m_n\omega t\ dt \\ \frac{\omega}{\pi} \int_0^T p(t) \sin m_n\omega t\ dt \end{array} \right\} \tag{8}$$

Remarks

- Equations (7) and (8) demonstrate the time-frequency character of the MHB algorithm where it is generally much easier to evaluate the forces in the time domain and then transform back into the frequency domain.
- Equation (5) is generally solved using a predictor-corrector continuation scheme in order to follow the distorsions of the corresponding frequency responses [12].
- Model reduction can be used effectively for the linear system matrices in order to reduce the computational burden for very large models.

2.3 Extended Constitutive Relation Error

The Constitutive Relation Error was initially proposed by Ladavèze et al. in the early 1980s as an error estimator for finite element models [13]. An extended version for use in model updating was described in the early 1990s [14] taking into account both modeling error and test-analysis errors for linear elastodynamic behaviors. A discrete formulation of the approach for dissipative linear structures can be found in [15]. The basic philosophy of the ECRE methodology consists in dividing the relations of interest (constitutive behavior laws, equations of motion, measured displacements, initial conditions, etc.) into two groups: the reliable and the less reliable quantities. The solution to the problem is sought so as to satisfy the reliable equations exactly while minimizing the errors in the less reliable equations. The present paper will be restricted to nonlinear elastodynamic systems which contain only nonlinear stiffness errors. Extensions to nonlinear dissipative effects as well as combined errors in both linear and nonlinear properties can be formulated in an analogous manner.

Let $\mathcal{Q}_\omega$ and $\mathcal{V}_\omega$ be two admissible displacement fields of equation (5) and $D^2(\mathcal{Q}_\omega, \mathcal{V}_\omega)$ a measure of distance between the two vectors such that:

$$D^2(\mathcal{Q}_\omega, \mathcal{V}_\omega) = \|\mathcal{Q}_\omega - \mathcal{V}_\omega\|^2_{\mathcal{K}} \equiv (\mathcal{Q}_\omega - \mathcal{V}_\omega)^T \mathcal{K} (\mathcal{Q}_\omega - \mathcal{V}_\omega) \tag{9}$$

where, $\mathcal{K} \in \Re(2n+1)N, (2n+1)N$ is the multi-harmonic stiffness matrix corresponding to the linear system defined by:

$$\mathcal{K} = \begin{bmatrix} K & 0 & 0 & \cdots & 0 & 0 \\ 0 & K & 0 & \cdots & 0 & 0 \\ 0 & 0 & K & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & K & 0 \\ 0 & 0 & 0 & \cdots & 0 & K \end{bmatrix} \tag{10}$$

A multi-harmonic ECRE can be defined for nonlinear stiffness errors in the following way:

$$E^2_\omega = r^T_\omega \mathcal{K} r_\omega + \alpha \left(H\mathcal{Q}_\omega - \mathcal{Q}^e_\omega\right)^T \mathcal{K}_R \left(H\mathcal{Q}_\omega - \mathcal{Q}^e_\omega\right) \tag{11}$$

where,

- $r_\omega \equiv \mathcal{Q}_\omega - \mathcal{V}_\omega$, with $\mathcal{Q}_\omega \in \Re^{(2n+1)N,1}$ and $\mathcal{V}_\omega \in \Re^{(2n+1)N,1}$ two admissible displacement fields for multi-harmonic equation of motion equation (5).
- $\mathcal{Q}^e_\omega \in \Re^{(2n+1)n_e,1}$ is the vector of identified harmonic coefficients on the n_e measurement degrees-of-freedom.
- $H \in \Re^{(2n+1)n_e,(2n+1)N}$ is a projection matrix allowing the model responses $\mathcal{Q}_\omega$ to be projected onto the set of n_e measurement directions so as to account for the limited number of measurement dofs and any differences in local reference frames between the FE model and the experimental model.
- $\mathcal{K}_R \in \Re^{(2n+1)n_e,(2n+1)n_e}$ is the multi-harmonic stiffness matrix of the linear system reduced to the measurement degrees-of-freedom. In practice, the Guyan stiffness matrix is generally used.
- α is a real positive scalar allowing the relative confidence in the identified harmonic coefficients to be taken into account.

Equation (11) is composed of two terms. The first term is a measure of the modeling error whereas the second term is a measure of the distance between the experimentally identified harmonic coefficients and those predicted by the model. Both of these terms correspond to the less reliable quantities in the present ECRE formulation. The reliable quantities correspond to the equilibrium equations of the system expressed by equation (5).

Therefore, the minimization problem to be solved in this case is given by:

$$\begin{cases} \text{Minimize} & E^2_\omega = r^T_\omega \mathcal{K} r_\omega + \alpha \|H\mathcal{Q}_\omega - \mathcal{Q}^e_\omega\|^2_{\mathcal{K}_R} \\ \text{Under the constraint} & \mathcal{K} r_\omega = \mathcal{Z}(\omega)\mathcal{Q}_\omega + \mathcal{F} - \mathcal{P} \end{cases} \tag{12}$$

or again:

$$\min g = r_\omega^T \mathcal{K} r_\omega + \alpha(H\mathcal{Q}_\omega - \mathcal{Q}_\omega^e)^T \mathcal{K}_R (H\mathcal{Q}_\omega - \mathcal{Q}_\omega^e) + \gamma^T(\mathcal{K} r_\omega - Z(\omega)\mathcal{Q}_\omega - \mathcal{F} + \mathcal{P}) \tag{13}$$

where g is the objective function and $\gamma \in \Re^{(2n+1)N,1}$ is a vector of Lagrange multipliers.

The stationarity conditions require:

$$\begin{aligned}
\frac{\partial g}{\partial r_\omega} = 0 \quad &\Rightarrow \quad \mathcal{K}(2r_\omega + \gamma) = 0 \\
\frac{\partial g}{\partial Q_\omega} = 0 \quad &\Rightarrow \quad -\mathcal{Z}(\omega)\gamma - \frac{\partial \mathcal{F}}{\partial \mathcal{Q}_\omega}\gamma + 2\alpha H^T \mathcal{K}_R (H\mathcal{Q}_\omega - \mathcal{Q}_\omega^e) = 0 \\
\frac{\partial g}{\partial \gamma} = 0 \quad &\Rightarrow \quad \mathcal{K} r_\omega - \mathcal{Z}(\omega)\mathcal{Q}_\omega - \mathcal{F} + \mathcal{P} = 0
\end{aligned} \tag{14}$$

Eliminating γ and regrouping the equations yields the following nonlinear matrix equation:

$$\begin{bmatrix} \mathcal{Z}(\omega) + \dfrac{\partial \mathcal{F}}{\partial \mathcal{Q}_\omega} & \alpha H^T \mathcal{K}_R H \\ \mathcal{K} & -Z(\omega) \end{bmatrix} \begin{Bmatrix} r_\omega \\ \mathcal{Q}_\omega \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\mathcal{F} \end{Bmatrix} = \begin{Bmatrix} \alpha H^T \mathcal{K}_R \mathcal{Q}_\omega^e \\ -\mathcal{P} \end{Bmatrix} \tag{15}$$

Remarks

- Equation (15) requires the solution of a nonlinear system of order $2N(2n+1)$. It can be solved with a classical Newton-Raphson iterative procedure.
- The solution of equation (15) comprises two unknown vectors. First, the residual vector r_ω represents the displacement field resulting from the unbalanced forces in the multi-harmonic equations of motion and provides the basis for calculating the modeling error. Second, the response vector $\mathcal{Q}_\omega$ represents the experimental multi-harmonic response expanded onto all model dofs and provides a means for evaluating the test-analysis distances.
- Given the vectors r_ω and $\mathcal{Q}_\omega$, the total MHB-ECRE error equation (11) for the point in model space defined by the nominal linear system matrices and the nominal nonlinear model used to estimate the multi-harmonic nonlinear forces can now be evaluated.
- The model updating problem finally consists in minimizing the total MHB-ECRE error over the space defined by coefficients of the nonlinear model.
- $\mathcal{Q}_\omega^e$ represents the vector of experimentally identified harmonic coefficients. It is obtained directly from the experimentally observed time responses via the Fast Fourier Transform (FFT) [16]:

$$\mathcal{Q}_j^c = \frac{1}{N_p} \sum_{k=0}^{N_p-1} q(k) \, \cos\left(\frac{2\pi}{N_p} kj\right) \tag{16}$$

$$\mathcal{Q}_j^s = \frac{-i}{N_p} \sum_{k=0}^{N_p-1} q(k) \, \sin\left(\frac{2\pi}{N_p} kj\right) \tag{17}$$

  where, N_p is the number of points per period and i is the imaginary number.

- Model reduction can again be effectively used here to minimize memory requirements and calculation times.

3 Example

The proposed methodology will be illustrated on a simulated academic example based on the COST action F3 project benchmark [17]. The model consists in a clamped linear beam attached to a thinner beam at one end. The main beam has a length of 0.7 m and a thickness of 0.014 m, whereas the thin beam has a length of 0.04 m with a thickness of 0.0005 m. Both beams have a width of 0.014 m and the material of both of them is steel with a Young's modulus of 210 GPa and a Poisson ratio of 0.3. The structure is excited at node number 3 (see Figure 1) with a stepped sine excitation having an amplitude of 2. This amplitude level was chosen based on the results of [3] in order to insure a large enough deflection for nonlinear effects to come into play.

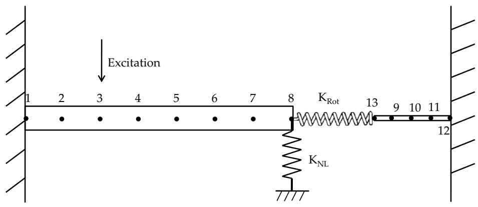
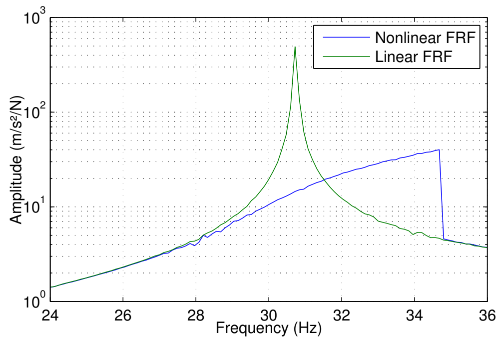


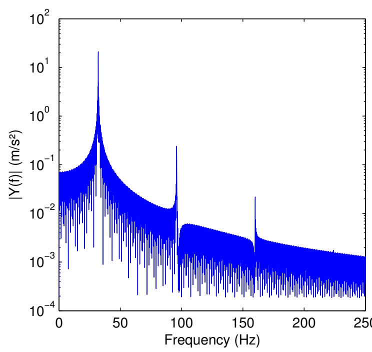
Figure 1: CostF3 beam

As stated in [3] and [18], the nonlinear behavior appears mainly in the first mode (30.76 Hz). A grounded cubic stiffness is introduced at node number 8 with the goal of modeling these nonlinearities. Node number 13 is placed in the same location as node 8. These two nodes are constrained to have the same translational displacements while a rotational spring is placed between their in-plane rotational dof. Moreover, in this example the influence of this cubic nonlinearity is studied only for the first mode. The nominal value of the nonlinear coefficient was chosen to be $6.1\ 10^9$ N/m [18].

The FRF is calculated between the excitation point (node 3) and the response point (node 8) and plotted in Figure 2(a) in order to visualize the distortion resulting from the nonlinear effects. Figure 2(b) displays the FFT of the time response of node 8 to a 32 Hz sine excitation. A peak at the fundamental frequency is observed as well as the 3^{rd} (96 Hz) and 5^{th} (160 Hz) harmonics. The 7^th harmonic (224 Hz) is also present but barely visible.



(a) FRF between the excitation point (node 3) and the response point (node 8)



(b) FFT of node 8 time response for a $32Hz$ excitation

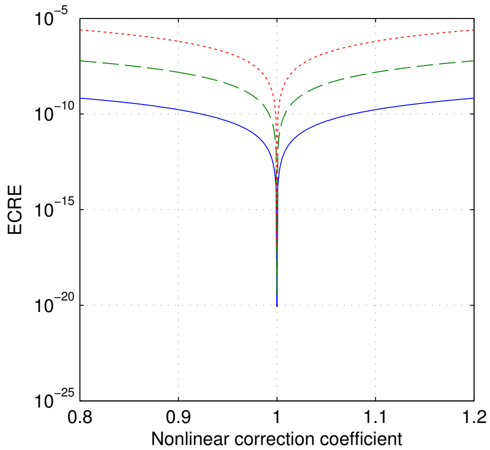
Figure 2: Frequency domain responses of the beam

In this example, the experimental vector of harmonic coefficients contained in $\mathcal{Q}^e_\omega$ are simulated numerically using a Newmark algorithm based on [19] followed by a FFT analysis.

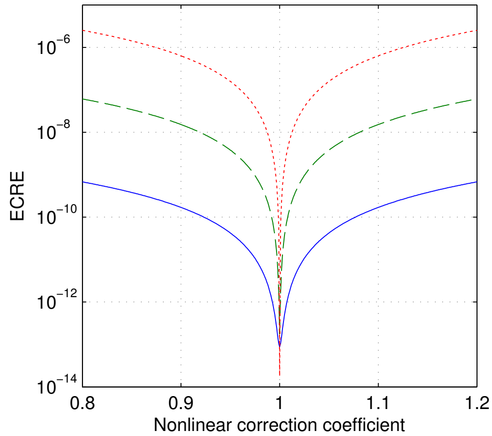
In order to illustrate the advantages and limitations of the proposed nonlinear updating strategy, it will be applied in four different simulated test configurations. Three excitation frequencies will be investigated corresponding to different response levels and thus different degrees of nonlinearity. The objective here is simply to examine the shape of the error expressed by equation (11) as a function of a single nonlinear model parameter. To simplify the interpretation of the results, the experimental harmonic coefficients have been generated based on the nominal nonlinear model. As such, in what follows it is expected to see a minimum in the MHB-ECRE curve at a value of the correction coefficient that multiplies the nonlinear stiffness (K_{NL}) equal to 1. The number of harmonics taken into account in the MHB-ECRE procedure will also be studied here.

Test case 1: Verification of the implemented algorithm

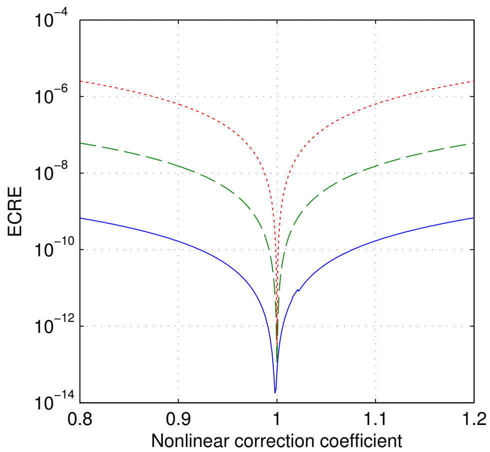
The objective of this first configuration is simply to verify the implemented MHB-ECRE algorithm. In this case, all model degrees of freedom (dofs) have been measured, that is to say, all 21 dofs (10 translations and 11 rotations) of the beam in Figure 1 are observed. Figure 3 plots the results of the MHB-ECRE updating procedure. In the case where the fundamental, the 3^{rd}, 5^{th} and 7^{th} harmonics are taken into account, in Figure 3(a), the MHB-ECRE curves are, as expected, minimum for a correction coefficient equal to 1. In Figures 3(b) and 3(c), where the 7^{th} and $7^{th}+5^{th}$ harmonics are respectively removed from the MHB-ECRE calculation, the results still give a good estimation of the nonlinear parameter. In the case where only the fundamental contribution is retained, in Figure 3(d), the procedure is still accurate for frequencies 32 Hz and 30 Hz, whereas for 28 Hz the minimum value is slightly overestimated. It can be noted that all three curves are convex, which is an advantage in finding the minimum of the MHB-ECRE function.



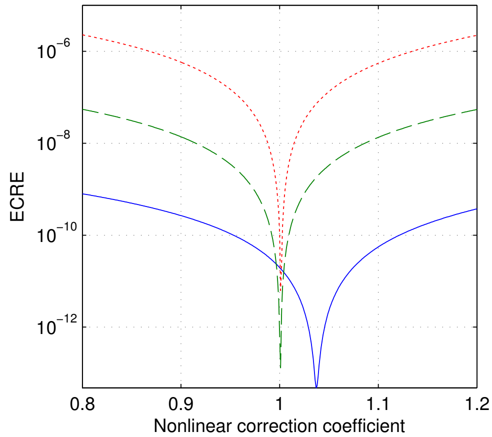
(a) Fundamental + H3 + H5 + H7



(b) Fundamental + H3 + H5



(c) Fundamental + H3

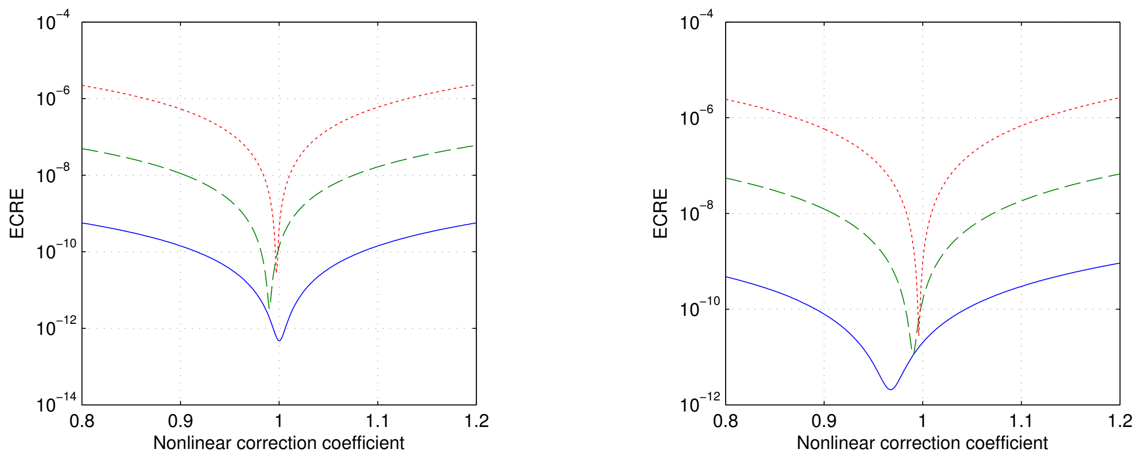


(d) Fundamental only

Figure 3: Complete model and 21 dofs measured : MHB-ECRE results
— 28 Hz, - - - 30 Hz, . . . 32 Hz

Test case 2: Impact of a reduced set of measurement dofs

The second case aims at illustrating the impact of observing only a subset of a model dofs. In the present case, only 4 translations are assumed to be measured corresponding to nodes 3, 4, 6 and 8. The model reduction has been performed based on the static Guyan procedure [20]. Finally, the model dof corresponding to the nonlinear cubic spring is assumed to be included in the set of observed dofs. The comparison between the results of the MHB-ECRE for the three different excitation frequencies, taking into account the fundamental contribution only and the fundamental plus the three first odd harmonics, are plotted in Figure 4(a) and Figure 4(b), respectively. The curves are still convex in both cases. However, in Figure 4(b), the nonlinear parameters are now underestimated although the MHB-ECRE curve for the highest excitation frequency (corresponding to the highest response amplitude and thus the largest nonlinear effect) still has a minimum at very nearly 1. These shifts are due to the fact that the Guyan reduction is no longer an exact representation of the dynamics of the linear system. However, as the nonlinear effects increase in magnitude, this discrepancy becomes less and less important. Moreover, a compensation effect between errors due to model reduction and the loss of information due to harmonic truncation can be observed in the results at 28 Hz. Indeed, the model reduction tends to shift the minimum to the left while the harmonic truncation tends to shift the minimum to the right.



(a) Fundamental only (b) Fundamental + H3 + H5 + H7

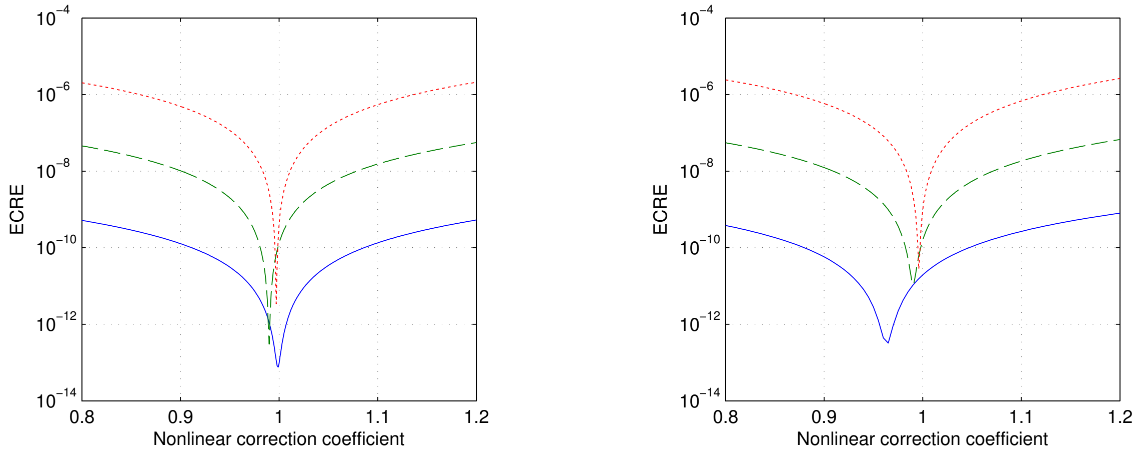
Figure 4: 4 dofs reduced model and 4 dofs measured : MHB-ECRE results
— 28 Hz, - - - 30 Hz, . . . 32 Hz

Test case 3: Impact of a lack of measurements on nonlinear dofs

This third case aims at illustrating a very important characteristic of the proposed updating strategy, namely that it is not necessary to experimentally observe the model degrees-of-freedom corresponding to the location of the nonlinear physics (the translational displacement at node 8 in this example). The same reduced system matrices as in the previous test case are retained, but it is assumed that the displacement of node 8 is no longer available. The results are plotted in Figure 5 and are qualitatively similar to those of the previous test case. That means that the measurement of the nonlinear dof is not required for an accurate estimation of the nonlinear coefficient.

Test case 4: Impact of the quality of the reduction model

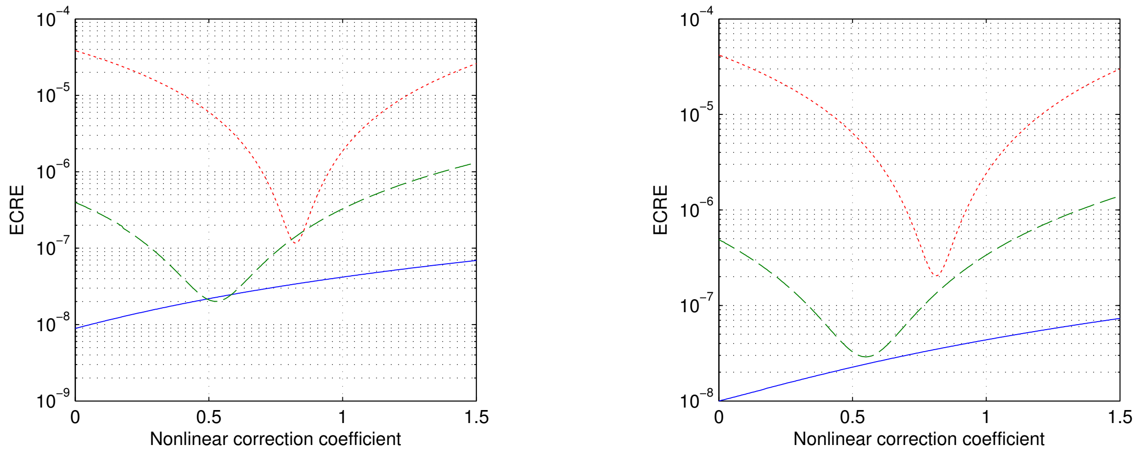
The goal of this last test case is to understand the shift in the MHB-ECRE curves observed between test case 1 and 2, that is to say, between a complete model and a reduced model. Indeed, an indicator is clearly required to quantify the relative impacts of model reduction errors and nonlinear effects. In this case, a 2



(a) Fundamental only (b) Fundamental + H3 + H5 + H7

Figure 5: 4 dofs reduced model and 3 dofs measured : MHB-ECRE results
— 28 Hz, - - - 30 Hz, . . . 32 Hz

dof reduced Guyan model is used corresponding to the translations at nodes 3 and 8. The results plotted in Figure 6 lead to an even larger shift in the minimums of the curves.



(a) Fundamental only (b) Fundamental + H3 + H5 + H7

Figure 6: 2 dofs reduced model and 2 dofs measured : MHB-ECRE results
— 28 Hz, - - - 30 Hz, . . . 32 Hz

The eigenfrequencies of the three different models are summarized in Table 1. Small differences can be noted for the 4 dofs reduced model: 0.02% and 0.5% of relative error for respectively the first and second modes. However, for the 2 dofs reduced model, the errors are more important: 1% for the first mode and 7.4% for the second. Moreover, the 5^{th} harmonic of the three excitation frequencies are 140 Hz, 150 Hz and 160 Hz, which is close to the second eigenfrequency. The poor accuracy of this reduced model for the first mode and even more on the second mode may explain the large shift observed in Figure 6.

	Complete model	4 dofs reduced model - relative error	2 dofs reduced model - relative error
mode 1	30.76 Hz	30.74 Hz - 0.02%	31.08 Hz - 1%
mode 2	150.62 Hz	151.35 Hz - 0.5%	161.72 Hz - 7.4%

Table 1: Eigenfrequencies and relative errors of the different studied models

Another way to understand the errors due to model reduction is to quantify the ratio between the residual error $\mathcal{R}$ of the reduced equilibrium equation (18) and the effective nonlinear force $\mathcal{F}$.

$$\mathcal{R} = \mathcal{Z}(\omega)\mathcal{Q} + \mathcal{F}(\mathcal{Q}) - \mathcal{P} \tag{18}$$

where, $\mathcal{Z} \in \Re(2n+1)N_r, (2n+1)N_r$ and $\mathcal{R}, \mathcal{F}, \mathcal{P} \in \Re(2n+1)N_r, 1$ and N_r is the number of dofs retained in the reduced model. Figure 7 plots the ratio $|\mathcal{R}|/|\mathcal{F}|$ calculated for the three frequencies, for the nominal value of the nonlinear parameter and taking into account all the harmonics. These results show that for a 2 dofs reduced model, the nonlinear information included in $\mathcal{F}$ is the same order of magnitude as the residual error and thus is not sufficient to have a good estimation of the non-linear parameter. However, for a 4 dof reduced model, and even more with 21 dofs (complete model), the ratio tends to 0. The nonlinear force is now more significant and, as shown in test case 1 & 2, the nonlinear coefficient can be effectively estimated.

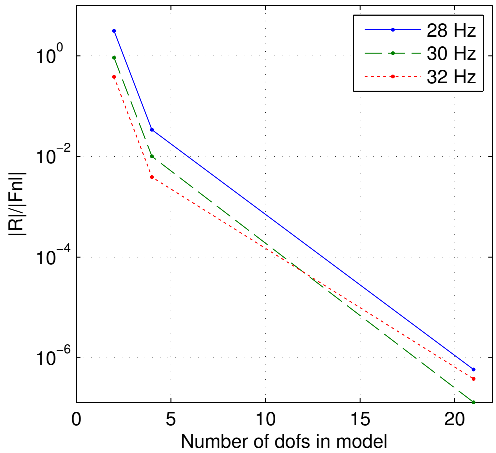


Figure 7: Influence of model reduction on results accuracy

In this case, a 2 dof static reduction is clearly insufficient and either more dofs must be included or an alternative model reduction technique must be used that takes into account the dynamic behavior of the slave structure, such as the Craig-Bampton [21] or Petersmann [22] methods.

4 Conclusions

This paper presents a novel nonlinear model updating approach that combines two well-known strategies for structural dynamic analysis, namely the Multi-harmonic Balance method for calculating the periodic response of a nonlinear system and the Extended Constitutive Relation Error method for establishing a well-behaved metric for modeling and test-analysis errors. The proposed updating strategy has been illustrated using simulated data based on the COST-F3 beam benchmark. The potential advantages of this methodology are:

- The absence of a locally linearized model thus allowing strongly nonlinear systems to be addressed with the inclusion of higher-order harmonics.
- It is not necessary to experimentally observe the structural displacements at the location of the non-linear physics.
- The model responses do not need to be re-evaluated at every updating iteration thus reducing the computational burden of the updating process.

- Model reduction techniques can be used very efficiently to reduce calculation costs, on condition that the reduced model is accurate enough at the frequencies of interest and at their harmonics.

The main limitation of the MHB-ECRE method concerns the impact of measurement noise and harmonic truncation effects on the total MHB-ECRE curves. A decision indicator is currently under investigation to quantify the level of nonlinearity that can reasonably be identified for a given level of model reduction and measurement uncertainty.

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